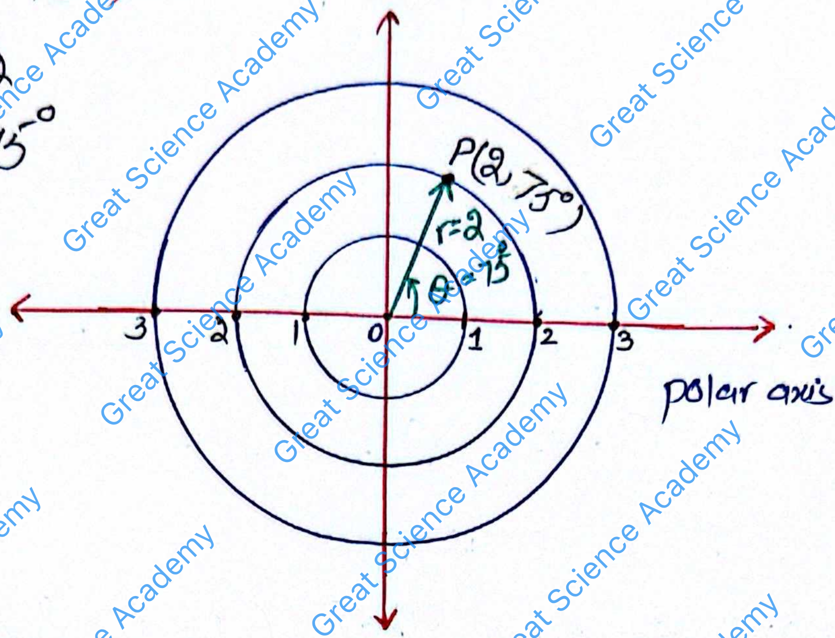
Exercise# 1.5

Q.1 Plot the following points.

i) $(2, 75^\circ)$

Let $|r| = 2$

$\theta = 75^\circ$



From fig the polar Co-ordinates $(2, 75^\circ)$ represent a point 2 units away from pole at angle of 75°

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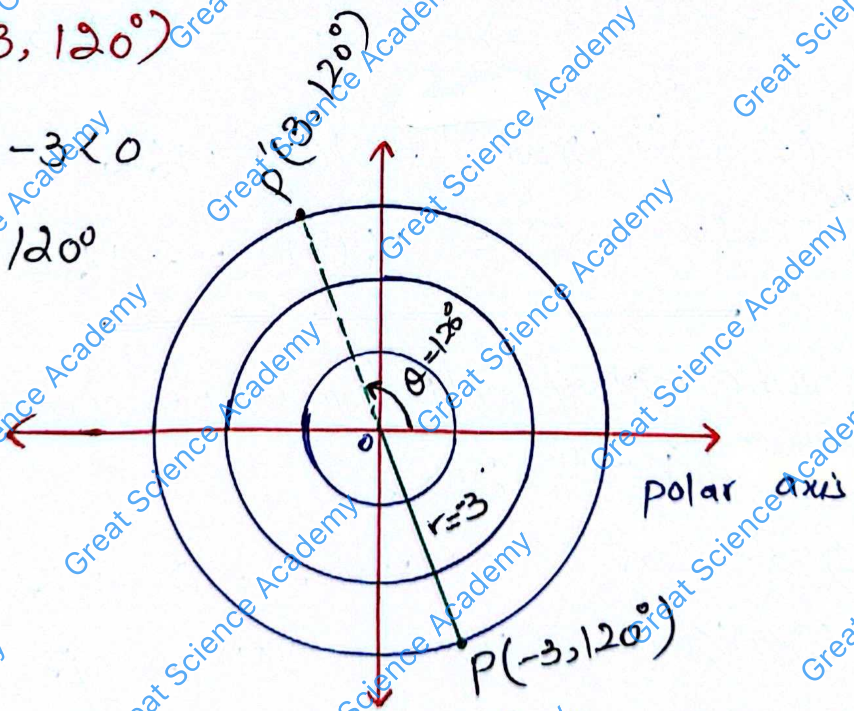
ii) $(-3, 120^\circ)$

Let $|r| = -3 < 0$

$\theta = 120^\circ$

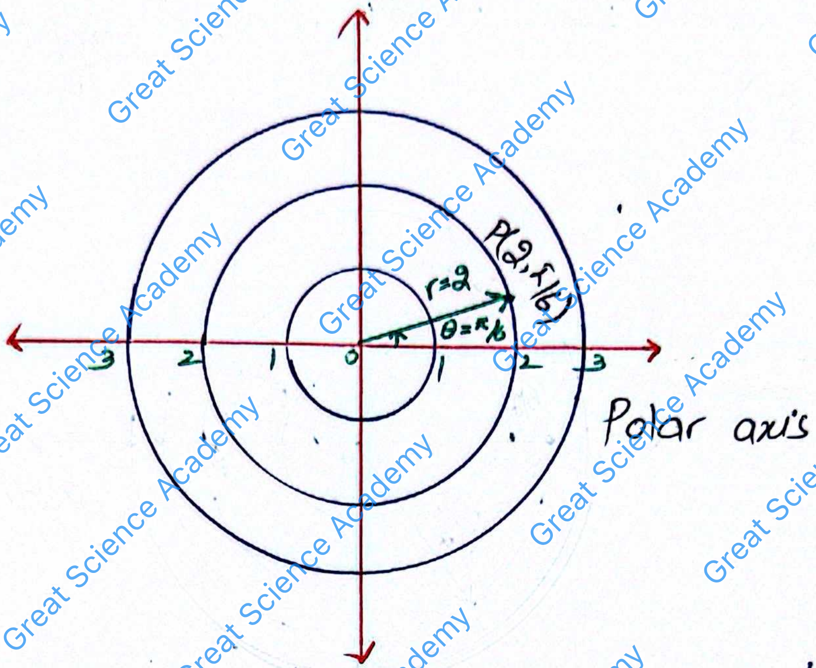


From fig the polar co-ordinates $(-3, 120^\circ)$ represent a point 3 unit away from pole but in the direction of 300°

iii) $(2, \pi/6)$

Let $r = 2 > 0$

$\theta = \pi/6$

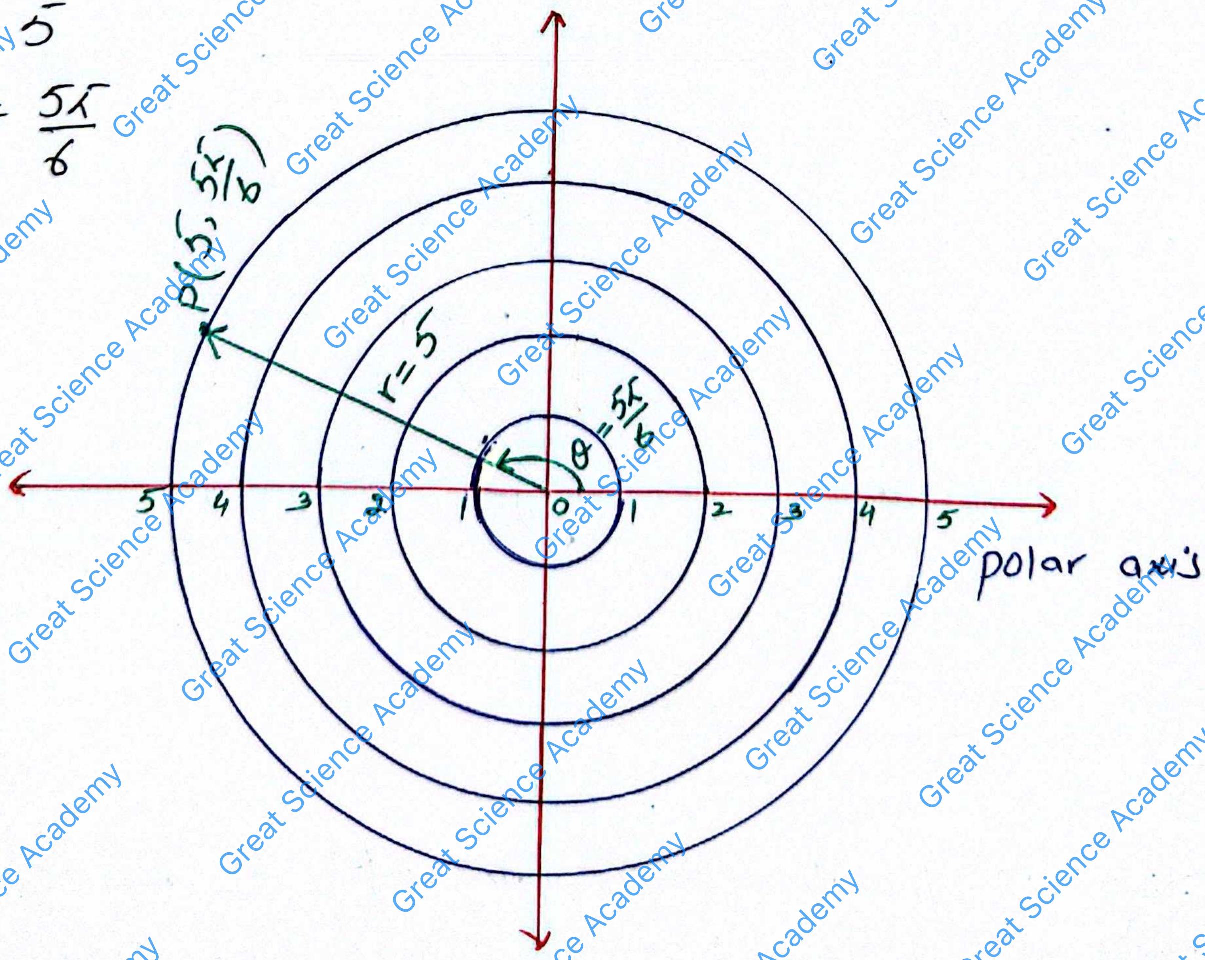


From fig the polar Co-ordinates $(2, \frac{\pi}{6})$ represent 2 unit away from the pole at angle $\pi/6$

iv) $(5, 5\pi/6)$

$r = 5$

$\theta = \frac{5\pi}{6}$



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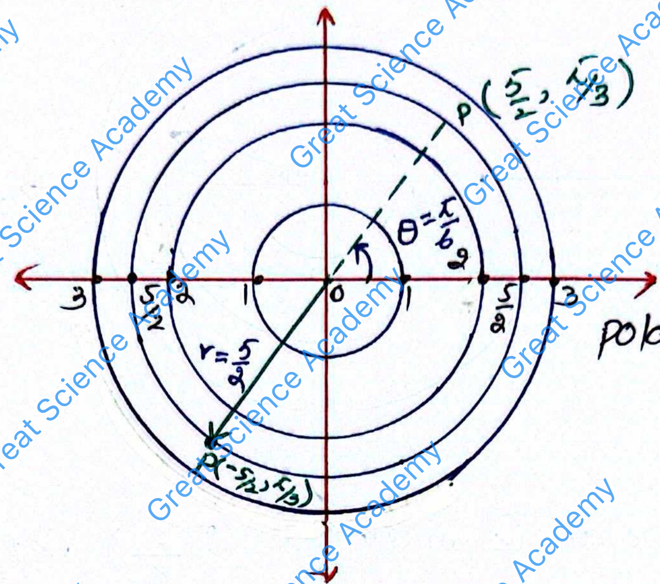
from fig the polar co-ordinates $(5, \frac{5\pi}{6})$ represent a point 5 unit from pole at the angle of $\frac{5\pi}{6}$.

v) $\left(-\frac{5}{2}, \frac{\pi}{3}\right)$

$|r| = -\frac{5}{2} < 0$

$\theta = \frac{\pi}{3}$



from fig the polar co-ordinates $(-\frac{5}{2}, \frac{\pi}{3})$ represent a point $\frac{5}{2}$ unit from the pole but in the direction of

$\frac{\pi}{3} + \pi = \frac{4\pi}{3}$ Radian.

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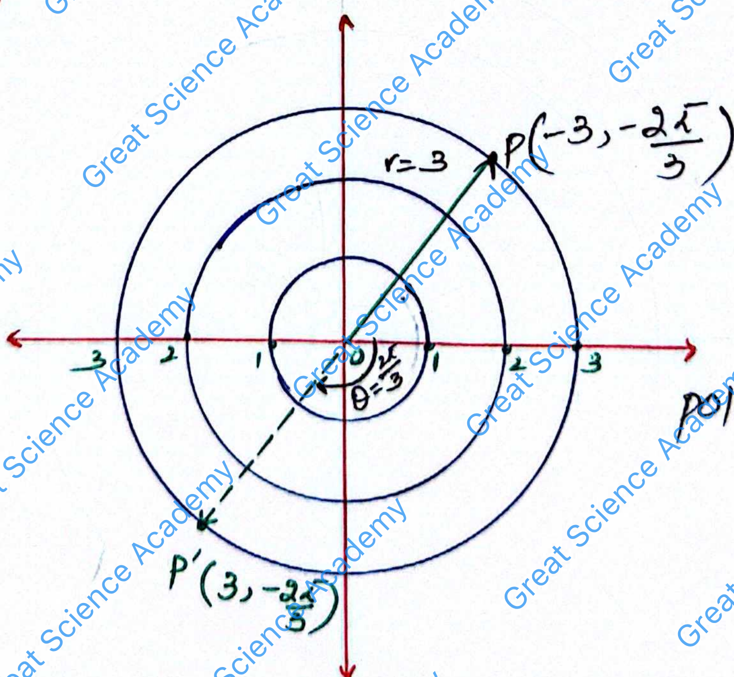
vi) $\left(-3, -\frac{2\pi}{3}\right)$

$|r| = -3 < 0$

$\theta = -\frac{2\pi}{3}$



from fig the the polar co-ordinates $(-3, -\frac{2\pi}{3})$ represent a point 3 unit from the pole but in the direction of

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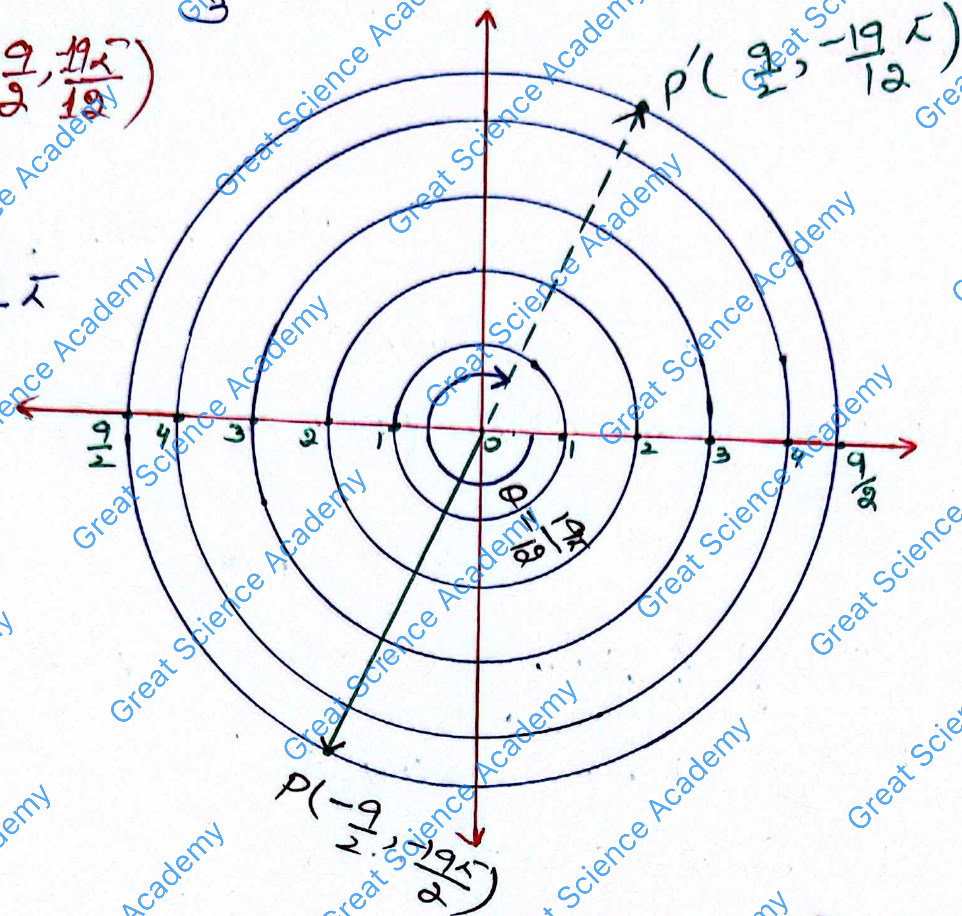
$-\frac{2\pi}{3} - \pi = -\frac{5\pi}{3}$ Radian.

viii) $\left(-\frac{9}{2}, \frac{19\pi}{12}\right)$

$|r| = -\frac{9}{2}$

$\theta = -\frac{19}{12}\pi$



from fig the polar co-ordinates $\left(-\frac{9}{2}, -\frac{19\pi}{12}\right)$ represent a point $\frac{9}{2}$ unit away from pole but in the direction of angle $\frac{7\pi}{12}$ Radian.

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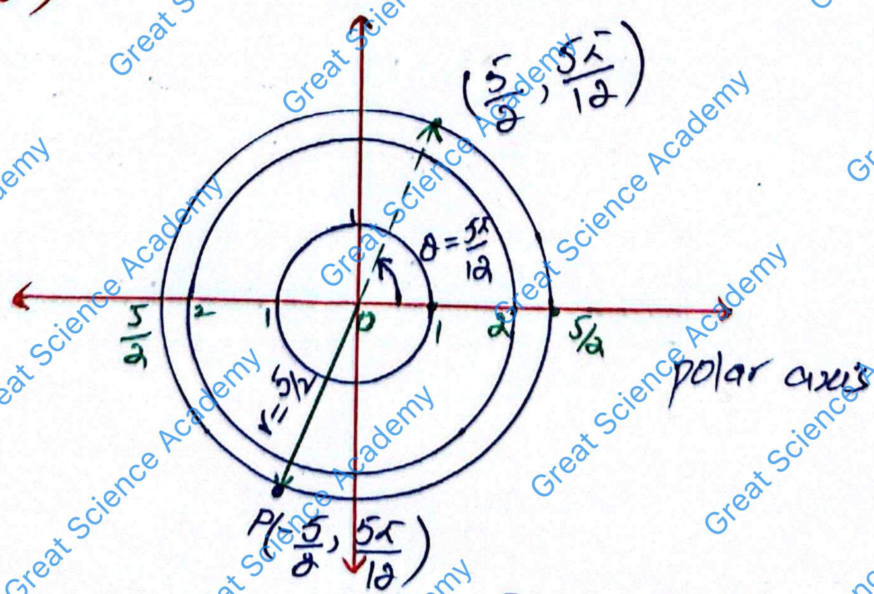
viii) $\left(-5/2, \frac{5\pi}{12}\right)$



from fig the polar co-ordinates $\left(-\frac{5}{2}, \frac{5\pi}{12}\right)$ represent a point $\frac{5}{2}$ unit away from the pole but in direction of angle $17\pi/12$.

Q.2 Express the following Complex number in polar form

i) $4+3i$

Sol:- Let $Z = 4+3i$ —— (1

The polar form of Complex number is

$Z = r\cos\theta + ir\sin\theta$ —— (2

Comparing equation (1 & (2

$r\cos\theta = 4$ —— (i $r\sin\theta = 3$ —— (ii

Squaring & adding equation (i & (ii we get

$r^2\cos^2\theta + r^2\sin^2\theta = 4^2 + 3^2$

$r^2(\sin^2\theta + \cos^2\theta) = 16+9$

$r^2(1) = 25 \Rightarrow r^2 = 25 \Rightarrow \boxed{r = 5}$

$\div$ing eq. (ii & (i we get

$\frac{r\sin\theta}{r\cos\theta} = \frac{3}{4} \Rightarrow \tan\theta = 3/4 \Rightarrow \theta = \tan^{-1}(3/4)$

$\boxed{\theta = 36.86^\circ}$

put $\theta = 36.86^\circ$ & $r = 5$ in equation (2

$Z = 5\cos(36.86^\circ) + 5i\sin(36.86^\circ)$

$\boxed{Z = 5(\cos(36.86^\circ) + i\sin(36.86^\circ))}$

ii) $1+i$

Sol:- Let $Z = 1+i$ —— (1

The polar of Complex number is

$Z = r\cos\theta + ri\sin\theta$ —— (2

Comparing equation (1 & (2) we get

$r\cos\theta = 1$ —— (i $\quad r\sin\theta = 1$ —— (ii

Squaring & adding eq. (i & (ii we get

$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + 1^2 \Rightarrow r^2(\sin^2\theta + \cos^2\theta) = 1+1$

$r^2(1) = 2 \Rightarrow r^2 = 2 \Rightarrow \boxed{r = \sqrt{2}}$

$\div$ing eq. (ii by (i we get

$\frac{r\sin\theta}{r\cos\theta} = \frac{1}{1} \Rightarrow \tan\theta = 1 \Rightarrow \theta = \tan^{-1}(1)$

$$\boxed{\theta = \pi/4}$$

put $r = \sqrt{2}$ & $\theta = \pi/4$ in eq. (2)

$Z = \sqrt{2}\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\sqrt{2}$

$\boxed{Z = \sqrt{2}\left[\cos(\pi/4) + i\sin(\pi/4)\right]}$ which is req.

iii) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Sol:- Let $Z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ —— (1

The polar form of Complex number is

$Z = r\cos\theta + ir\sin\theta$ —— (2

Comparing equation (1 & (2

$r\cos\theta = \frac{1}{2}$ —— (i $\quad r\sin\theta = \frac{\sqrt{3}}{2}$ —— (ii

Adding & Squaring eq (i & (ii we get

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$$r^2\cos^2\theta + r^2\sin^2\theta = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow r^2(\sin^2\theta + \cos^2\theta) = \frac{1}{4} + \frac{3}{4}$$

$$r^2(1) = \frac{1+3}{4} = \frac{4}{4} \Rightarrow r^2 = 1 \Rightarrow \boxed{r = 1}$$

$\div$ing (ii by (i we get

$$\frac{r\sin\theta}{r\cos\theta} = \frac{\sqrt{3}/2}{1/2} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \tan^{-1}(\sqrt{3})$$

$$\boxed{\theta = \frac{\pi}{3}}$$

put $r = 1$ & $\theta = \frac{\pi}{3}$ in eq. (2 we get

$$z = 1\cos(\pi/3) + i\sin(\pi/3)\cdot 1$$

$$\boxed{z = \cos(\pi/3) + i\sin(\pi/3)}$$ which is req.

iv) $-\frac{5}{2} - \frac{5\sqrt{3}}{2}i$

Sol:- Let $z = -\frac{5}{2} - \frac{5\sqrt{3}}{2}i$ —— (1

The polar form of complex number is

$$z = r\cos\theta + ir\sin\theta \text{ —— (2}$$

Comparing eq. (1 & (2 we get

$$r\cos\theta = -\frac{5}{2} \text{ —— (i} \qquad r\sin\theta = -\frac{5\sqrt{3}}{2} \text{ —— (ii}$$

Squaring & adding eq. (i & (ii we get

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\frac{5}{2}\right)^2 + \left(-\frac{5\sqrt{3}}{2}\right)^2$$

$$r^2(\sin^2\theta + \cos^2\theta) = \frac{25}{4} + \frac{25(3)}{4} = \frac{25}{4} + \frac{75}{4}$$

$$r^2 = \frac{25+75}{4} = \frac{100}{4} = 25 \Rightarrow \boxed{r = 5}$$

$\div$ing eq. (ii by eq. (i we get

$$\frac{r\sin\theta}{r\cos\theta} = \frac{-5\sqrt{3}/2}{-5/2} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \tan^{-1}(\sqrt{3})$$

$$\boxed{\theta = \pi/3}$$

As $Z = -\frac{5}{2} - \frac{5\sqrt{3}}{2}i$ lie in Q-III, then

Req. Angle $\theta = \pi + \theta = \frac{\pi}{1} + \frac{\pi}{3} = \frac{3\pi + \pi}{3}$

$$\boxed{\theta = \frac{4\pi}{3}}$$

put $r = 5$ & $\theta = \frac{4\pi}{3}$ in eq. (2 we get

$$Z = 5\cos\left(\frac{4\pi}{3}\right) + 5i\sin\left(\frac{4\pi}{3}\right)$$

$$\boxed{Z = 5\left[\cos\left(\frac{4\pi}{3}\right) + i\sin(4\pi/3)\right]}$$ which is req.

v) $-\frac{5}{7} + \frac{\sqrt{8}}{7}i$

let $Z = -\frac{5}{7} + \frac{\sqrt{8}}{7}i$ ——— (1

The polar form of Complex number is

$$Z = r\cos\theta + ir\sin\theta \text{ ——— (2}$$

Comparing eq. (1 & (2 we get

$r\cos\theta = -\frac{5}{7}$ ——— (i $\quad r\sin\theta = \frac{\sqrt{8}}{7}$ ——— (ii

Squaring & adding eq. (i & (ii we get

$$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\frac{5}{7}\right)^2 + \left(\frac{\sqrt{8}}{7}\right)^2$$

$$r^2(\cos^2\theta + \sin^2\theta) = \frac{25}{49} + \frac{8}{49} \Rightarrow r^2 = \frac{25+8}{49}$$

$$r^2 = \frac{33}{49} \Rightarrow \boxed{r = \frac{\sqrt{33}}{7}}$$

$\div$ing eq. (ii by (i we get

$$\frac{r\sin\theta}{r\cos\theta} = \left|\frac{\sqrt{8}/7}{-5/7}\right| \Rightarrow \tan\theta = \frac{\sqrt{8}}{5} \Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{8}}{5}\right)$$

$$\boxed{\theta = 19^\circ 28'} \text{ or } \boxed{\theta = 19.47^\circ}$$

As $z = -\frac{5}{7} + \frac{\sqrt{8}}{7}i$ lie in Q-II, then

Req. angle $\theta = \pi - \theta = \pi - 19.47^\circ = 180^\circ - 19.47^\circ$

$$\boxed{\theta = 160.53^\circ}$$

put $r = \frac{\sqrt{33}}{7}$ & $\theta = 160.53^\circ$ in eq. (2

$$z = \frac{\sqrt{33}}{7}\cos(160.53^\circ) + i\,\frac{\sqrt{33}}{7}\sin(160.53^\circ)$$

$$\boxed{z = \frac{\sqrt{33}}{7}\left[\cos(160.53^\circ) + i\sin(160.53^\circ)\right]}$$

vi) $\frac{1}{2} + \frac{\sqrt{5}}{2}i$

Sol:- Let $z = \frac{1}{2} + \frac{\sqrt{5}}{2}i$ —— (1

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The polar form of Complex number

$Z = r\cos\theta + i r\sin\theta$ —— (2

Comparing eq. (2 & (1 we get

$r\cos\theta = \frac{1}{2}$ —— (i $\quad r\sin\theta = \frac{\sqrt{5}}{2}$ —— (ii

Squaring & add eq. (i & (ii we get

$r^2\cos^2\theta + r^2\sin^2\theta = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{5}}{2}\right)^2 = \frac{1}{4} + \frac{5}{4}$

$r^2(1) = \frac{1+5}{4} = \frac{6}{4} \Rightarrow \boxed{r = \frac{\sqrt{6}}{2}} = \frac{\sqrt{2}\cdot\sqrt{3}}{2}$ OR

$r = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \boxed{r = \sqrt{\frac{3}{2}}}$

$\div$ing eq. (ii by (i we get

$\frac{r\sin\theta}{r\cos\theta} = \frac{\sqrt{5}/2}{1/2} \Rightarrow \tan\theta = \sqrt{5}$

$\Rightarrow \theta = \tan^{-1}(\sqrt{5}) \Rightarrow \boxed{\theta = 65.90^\circ}$

put $r = \frac{\sqrt{6}}{2}$ & $\theta = 65.90^\circ$ in eq. (2 we get

$$\boxed{Z = \frac{\sqrt{6}}{2}\left(\cos(65.90^\circ) + i\sin(65.90^\circ)\right)}$$

vii) $-\frac{2}{3} - \frac{\sqrt{7}}{3}i$

Sol:- Let $Z = -\frac{2}{3} - \frac{\sqrt{7}}{3}i$ —— (1

The polar form of the Complex number

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$z = r\cos\theta + i r\sin\theta$ —— (2

Comparing eq. (1 & (2 we get

$r\cos\theta = -\frac{2}{3}$ —— (i $\quad r\sin\theta = -\frac{\sqrt{7}}{3}$ —— (ii

Squaring & adding eq. (i & (ii we get

$r^2\cos^2\theta + r^2\sin^2\theta = \left(-\frac{2}{3}\right)^2 + \left(-\frac{\sqrt{7}}{3}\right)^2$

$r^2(\sin^2\theta + \cos^2\theta) = \frac{4}{9} + \frac{7}{9}$

$r^2 = \frac{4+7}{9} = \frac{11}{9} \Rightarrow \boxed{r = \frac{\sqrt{11}}{3}}$

$\div$ing eq. (ii by (i we get

$\frac{r\sin\theta}{r\cos\theta} = \frac{-\sqrt{7}/3}{-2/3} = \frac{\sqrt{7}}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{7}}{2}\right)$

$\theta = 52.91^\circ$

As $z = -\frac{2}{3} - \frac{\sqrt{7}}{3}$ lie in Q-III, then

Req. $\theta = \pi + 52.91^\circ = 180^\circ + 52.91^\circ$

$\boxed{\theta = 232.91^\circ}$

put $\theta = 232.91^\circ$ & $r = \frac{\sqrt{11}}{3}$ in eq. (2

$z = \frac{\sqrt{11}}{3}\cos(232.91^\circ) + i\frac{\sqrt{11}}{3}\sin(232.91^\circ)$

$\boxed{z = \frac{\sqrt{11}}{3}\left[\cos(232.91^\circ) + i\sin(232.91^\circ)\right]}$

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Q.3 Convert each of the following complex number z in the rectangular form $x+iy$.

i) $4\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$

Sol:- Let $z=4\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)$ —— (1

$\because \cos\frac{5\pi}{3}=\cos\left(2\pi-\frac{\pi}{3}\right)=\cos\left(4\times\frac{\pi}{2}-\frac{\pi}{3}\right)$

$\cos\frac{5\pi}{3}=\cos\frac{\pi}{3} \Rightarrow \boxed{\cos\frac{5\pi}{3}=\frac{1}{2}}$

$\because \sin\frac{5\pi}{3}=\sin\left(2\pi-\frac{\pi}{3}\right)=\sin\left(4\times\frac{\pi}{2}-\frac{\pi}{3}\right)$

$\sin\frac{5\pi}{3}=-\sin\frac{\pi}{3} \Rightarrow \boxed{\sin\frac{5\pi}{3}=-\frac{\sqrt{3}}{2}}$

put $\cos\frac{5\pi}{3}=\frac{1}{2}$ & $\sin\frac{5\pi}{3}=-\frac{\sqrt{3}}{2}$ in eq. (1

$z=4\left(\frac{1}{2}+i\left(-\frac{\sqrt{3}}{2}\right)\right)=4\cdot\frac{1}{2}-4i\cdot\frac{\sqrt{3}}{2}$

$\boxed{z=2-2\sqrt{3}i}$ Req. rectangular form.

ii) $\frac{3}{2}\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right)$

Sol:- Let $z=\frac{3}{2}\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right)$

$\because \cos\frac{7\pi}{6}=\cos\left(\pi+\frac{\pi}{6}\right)=\cos\left(2\times\frac{\pi}{2}+\frac{\pi}{6}\right)$

$=-\cos\frac{\pi}{6} \Rightarrow \boxed{\cos\frac{7\pi}{6}=-\frac{\sqrt{3}}{2}}$

$$\because \sin\frac{7\pi}{6} = \sin\left(\pi+\frac{\pi}{6}\right) = \sin\left(2\times\frac{\pi}{2}+\frac{\pi}{6}\right)$$

$$\sin\frac{7\pi}{6} = -\sin\frac{\pi}{6} \Rightarrow \boxed{\sin\frac{7\pi}{6} = -\frac{1}{2}}$$

put $\cos\frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ & $\sin\frac{7\pi}{6} = -\frac{1}{2}$ in eq. 2

$$Z = \frac{3}{2}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)$$

$$\boxed{Z = -\frac{3\sqrt{3}}{4} - \frac{3}{4}i}$$ Req. Rectangular form

iii) $|Z| = 7$ $\arg(z) = \frac{23\pi}{12}$

Sol:- Given that

$$|Z| = r = 7 \quad \theta = \arg(z) = \frac{23\pi}{12}$$

The polar form complex number Z is

$$Z = r(\cos\theta + i\sin\theta)$$

$$Z = 7\left(\cos\frac{23}{12}\pi + i\sin\frac{23}{12}\pi\right) \text{———} (1)$$

$$\because \cos\frac{23}{12}\pi = \cos\left(2\pi - \frac{\pi}{12}\right) = \cos\left(4\times\frac{\pi}{2} - \frac{\pi}{12}\right)$$

$$\cos\frac{23}{12}\pi = \cos\frac{\pi}{12} \Rightarrow \boxed{\cos\frac{23}{12}\pi = \frac{\sqrt{6}+\sqrt{2}}{4}}$$

$$\sin\frac{23\pi}{12} = \sin\left(2\pi - \frac{\pi}{12}\right) = \sin\left(4\times\frac{\pi}{2} - \frac{\pi}{12}\right)$$

$$\sin\frac{23\pi}{12} = -\sin\frac{\pi}{12} \Rightarrow \boxed{\sin\frac{23\pi}{12} = -\left(\frac{\sqrt{6}-\sqrt{4}}{4}\right)}$$

put $\cos\frac{23}{12}\pi = \frac{\sqrt{6}+\sqrt{2}}{4}$ & $\sin\frac{23\pi}{12} = -\left(\frac{\sqrt{6}-\sqrt{4}}{4}\right)$ in (1

$$Z = 7\left[\cos\frac{23}{12}\pi + i\sin\frac{23}{12}\pi\right]$$

$$Z = 7\left[\frac{\sqrt{6}+\sqrt{2}}{4} + i\left(-\frac{\sqrt{6}-\sqrt{4}}{4}\right)\right]$$

$$\boxed{Z = 7\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - 7\left(\frac{\sqrt{6}-\sqrt{4}}{4}\right)i}$$ which is Req.

(iv) $|Z| = 11$, $\arg(z) = -\frac{11\pi}{12}$

Sol:- Given that

$$|Z| = r = 11, \quad \theta = \arg(z) = -\frac{11}{12}\pi$$

The polar form of Complex number is

$$Z = r(\cos\theta + i\sin\theta)$$

$$Z = 11\left[\cos\left(-\frac{11\pi}{12}\right) + i\sin\left(-\frac{11\pi}{12}\right)\right]$$

$$Z = 11\left[\cos\frac{11\pi}{12} - i\sin\left(\frac{11\pi}{12}\right)\right]$$

$$Z = 11\left[-\frac{\sqrt{6}-\sqrt{2}}{4} - i\,\frac{\sqrt{6}-\sqrt{2}}{4}\right]$$

$$\boxed{Z = -11\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - 11\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)i}$$

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v) $|Z| = \frac{10}{3}$, $\arg(z) = -\frac{17\pi}{12}$

Sol:- Given that

$$|r| = |Z| = \frac{10}{3}, \quad \theta = \arg(z) = -\frac{17\pi}{12}$$

The polar form of the complex number is

$$Z = r(\cos\theta + i\sin\theta)$$

$$Z = \frac{10}{3}\left[\cos\left(-\frac{17\pi}{12}\right) + i\sin\left(-\frac{17\pi}{12}\right)\right]$$

$$Z = \frac{10}{3}\left[\cos\frac{17\pi}{12} - i\sin\frac{17\pi}{12}\right]$$

$$Z = \frac{10}{3}\left[-\frac{\sqrt{6}+\sqrt{2}}{4} - i\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right)\right]$$

$$Z = \frac{10}{3}\left(\frac{-\sqrt{6}+\sqrt{2}}{4}\right) + \frac{10}{3}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)i$$

$$Z = \frac{10(\sqrt{2}-\sqrt{6})}{12} + \frac{10(\sqrt{2}+\sqrt{6})}{12}i$$

$$\boxed{Z = \frac{5(\sqrt{2}-\sqrt{6})}{6} + \frac{5(\sqrt{2}+\sqrt{6})}{12}i}$$

vi) $2(\cos(-33)) + i\,2\sin(-33)$

Sol:- Let $Z = 2\cos(-33) + i\,2\sin(-33)$

$$Z = 2\cos 33 - i\,2\sin 33$$

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$$Z = 2(0.8387) + i\,2(0.5446)$$

$$\boxed{Z = (1.6773) - (1.0893)i}$$ which is req.

vii) $|Z| = 12$, $\arg(z) = \pi$

Sol:— Given that

$$r = |Z| = 12 \quad \theta = \operatorname{Arg}(z) = \pi$$

The polar form of Complex number is

$$Z = r\cos\theta + i r\sin\theta$$

$$Z = 12\cos(\pi) + i\,12\sin(\pi)$$

$$Z = 12(-1) + i\,12(0)$$

$$\boxed{Z = -12 + 0i}$$ which is req.

Q.4 If $Z_1 = 9\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$ & $Z_2 = 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ then find.

i) $Z_1 + Z_2 = ?$

$$Z_1 + Z_2 = 9\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) + 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= 9\left[-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right] + 5\left[\frac{1}{2} + \frac{\sqrt{3}}{2}i\right]$$

$$= -\frac{9\sqrt{2}}{2} - \frac{9\sqrt{2}}{2}i + \frac{5}{2} + \frac{5\sqrt{3}}{2}i$$

$$= \left(-\frac{9\sqrt{2}}{2} + \frac{5}{2}\right) + \left(\frac{5\sqrt{3}}{2} - \frac{9\sqrt{2}}{2}\right)i$$

$$\boxed{Z_1 + Z_2 = \left(\frac{5-9\sqrt{2}}{2}\right) + \left(\frac{5\sqrt{3}-9\sqrt{2}}{2}\right)i}$$ Ans

ii) $Z_1 - Z_2 = ?$

$$Z_1 - Z_2 = 9\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) - 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$= 9\left(-\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right) - 5\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$= -\frac{9\sqrt{2}}{2} - \frac{9\sqrt{2}}{2}i - \frac{5}{2} - \frac{5\sqrt{3}}{2}i$$

$$Z_1 - Z_2 = -\left(\frac{5}{2} + \frac{9\sqrt{2}}{2}\right) - \left(\frac{5\sqrt{3}}{2} + \frac{9\sqrt{2}}{2}\right)i$$

$$\boxed{Z_1 - Z_2 = -\left(\frac{5+9\sqrt{2}}{2}\right) - \left(\frac{5\sqrt{3}+9\sqrt{2}}{2}\right)i}$$ Ans

iii) $Z_1 \cdot Z_2 = ?$

The product of two Complex number in polar form is

$$Z_1 \cdot Z_2 = r_1 \cdot r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right] \text{ ——— } (1$$

As $Z_1 = 9\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$ $\wedge$ $Z_2 = 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\Rightarrow r_1 = 9, \ \theta_1 = \frac{5\pi}{4}$ $\wedge$ $r_2 = 5, \ \theta_2 = \frac{\pi}{3}$

put in eq. (1 we get we get

$$Z_1 \cdot Z_2 = 9 \cdot 5\left[\cos\left(\frac{5\pi}{4} + \frac{\pi}{3}\right) + i\sin\left(\frac{5\pi}{4} + \frac{\pi}{3}\right)\right]$$

$$Z_1 \cdot Z_2 = 45\left[\cos\left(\frac{15\pi+4\pi}{12}\right) + i\sin\left(\frac{15\pi+4\pi}{12}\right)\right]$$

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$$\boxed{Z_1 \cdot Z_2 = 45\left[\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right]}$$ Ans

(iv) $\dfrac{Z_1}{Z_2}$

The division of two complex number in polar form is

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right] \quad \text{——(1)}$$

As $Z_1 = \left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$, $Z_2 = 5\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$\Rightarrow \quad r_1 = 9 \quad \theta_1 = \frac{5\pi}{4} \quad \& \quad r_2 = 5, \quad \theta_2 = \frac{\pi}{3}$

putting the values in eq. (1) we get

$$\frac{Z_1}{Z_2} = \frac{9}{5}\left[\cos\left(\frac{5\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{5\pi}{4} - \frac{\pi}{3}\right)\right]$$

$$\frac{Z_1}{Z_2} = \frac{9}{5}\left[\cos\left(\frac{15\pi - 4\pi}{12}\right) + i\sin\left(\frac{15\pi - 4\pi}{12}\right)\right]$$

$$\boxed{\frac{Z_1}{Z_2} = \frac{9}{5}\left[\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right]}$$ Ans

Q.5 If $Z_1 = 7\left(\cos\frac{23}{12}\pi + i\sin\frac{23}{12}\pi\right)$ & $Z_2 = 11\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$ then the following and express the result into $x + iy$ form

i) $Z_1 + Z_2$

Given that

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$$Z_1 = 7\left(\cos\frac{23\pi}{12} + i\sin\frac{23}{12}\pi\right), \quad Z_2 = 11\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$$

$$Z_1 + Z_2 = 7\left(\cos\frac{23\pi}{12} + i\sin\frac{23}{12}\pi\right) + 11\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$$

$$= 7\left[\frac{\sqrt{6}+\sqrt{2}}{4} + i\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)\right] + 11\left[\frac{-\sqrt{6}-\sqrt{2}}{4} + i\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\right]$$

$$= 7\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + 7\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)i - 11\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - 11i\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)$$

$$= -4\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) - 4\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)i$$

$$\boxed{Z_1 + Z_2 = -(\sqrt{6}+\sqrt{2}) - (\sqrt{2}-\sqrt{6})i} \quad \text{Ans}$$

ii) $Z_1 - Z_2$

$$Z_1 - Z_2 = 7\left(\cos\frac{23}{12}\pi + i\sin\frac{23}{12}\pi\right) - 11\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$$

$$Z_1 - Z_2 = 7\left[\frac{\sqrt{6}+\sqrt{2}}{4} + i\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)\right] - 11\left[\frac{-\sqrt{6}-\sqrt{2}}{4} + i\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\right]$$

$$= 7\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + 7\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)i + 11\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + 11\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)i$$

$$= 18\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + 18\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)i$$

$$\boxed{Z_1 - Z_2 = \frac{9}{2}(\sqrt{6}+\sqrt{2}) + \frac{9}{2}(\sqrt{2}-\sqrt{6})i} \quad \text{Ans}$$

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iii) $Z_1 \cdot Z_2 = ?$

The product of two complex number in polar form

$$Z_1 \cdot Z_2 = r_1 \cdot r_2 \left[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2) \right] \text{ ——— } (1)$$

As $Z_1 = 7\left(\cos\frac{23\pi}{12} + i \sin\frac{23}{12}\pi\right)$ & $Z_2 = 11\left(\cos\frac{11\pi}{12} + i \sin\frac{11\pi}{12}\right)$

$r_1 = 7, \ \theta_1 = \frac{23\pi}{12}$ & $r_2 = 11, \ \theta_2 = \frac{11\pi}{12}$

putting the values of $r_1, r_2, \theta_1, \theta_2$ in eq. (1)

$$Z_1 \cdot Z_2 = 7 \cdot 11 \left[\cos\left(\frac{23\pi}{12} + \frac{11\pi}{12}\right) + i \sin\left(\frac{23}{12}\pi + \frac{11\pi}{12}\right) \right]$$

$$= 77 \left[\cos\left(\frac{23\pi + 11\pi}{12}\right) + i \sin\left(\frac{23\pi + 11\pi}{12}\right) \right]$$

$$= 77 \left[\cos\left(\frac{34\pi}{12}\right) + i \sin\left(\frac{34\pi}{12}\right) \right]$$

$$= 77 \left[-\frac{\sqrt{3}}{2} + \frac{i}{2} \right]$$

$$\boxed{Z_1 \cdot Z_2 = -\frac{77\sqrt{3}}{2} + \frac{77}{2} i}$$ Ans

iv) $\frac{Z_1}{Z_2} = ?$

The division of two complex number in polar form

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2} \left[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2) \right] \text{ ——— } (1)$$

As $Z_1 = 7\left(\cos\frac{23\pi}{12} + i \sin\frac{23\pi}{12}\right)$ & $Z_2 = 11\left(\cos\frac{11\pi}{12} + i \sin\frac{11\pi}{12}\right)$

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As $r_1 = 7$, $\theta_1 = \frac{23\pi}{12}$ & $r_2 = 11$, $\theta_2 = \frac{11\pi}{12}$

putting the values of r_1, θ_1, r_2 & θ_2 in eq. (2)

$$\frac{Z_1}{Z_2} = \frac{7}{11}\left[\cos\left(\frac{23\pi}{12} - \frac{11\pi}{12}\right) + i\sin\left(\frac{23}{12}\pi - \frac{11}{12}\pi\right)\right]$$

$$= \frac{7}{11}\left[\cos\left(\frac{23\pi - 11\pi}{12}\right) + i\sin\left(\frac{23\pi - 11\pi}{12}\right)\right]$$

$$= \frac{7}{11}\left[\cos\left(\frac{12\pi}{12}\right) + i\sin\left(\frac{12\pi}{12}\right)\right]$$

$$= \frac{7}{11}(\cos\pi + i\sin\pi) = \frac{7}{11}(-1 + 0i)$$

$$\boxed{\frac{Z_1}{Z_2} = \frac{-7}{11} + 0i}$$ Ans

Q.6 divide $Z_1 = 6(\cos 150° + i\sin 150°)$ by $Z_2 = 3(\cos 30° + i\sin 30°)$ and express in $x+iy$ form.

Sol:- Given that

$Z_1 = 6(\cos 150° + i\sin 150°)$ & $Z_2 = 3(\cos 30° + i\sin 30°)$

The division of Two Complex number in polar form

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

put $r_1 = 6$, $\theta_1 = 150°$ & $r_2 = 3$, $\theta_2 = 30°$

$$= \frac{6}{3}\left[\cos(150° - 30°) + i\sin(150° - 30°)\right]$$

$$\frac{Z_1}{Z_2} = 2\left[\cos 120^\circ + i\sin(120^\circ)\right]$$

$$= 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2\cdot\frac{1}{2}\left(-1+\sqrt{3}i\right)$$

$$\boxed{\frac{Z_1}{Z_2} = -1+\sqrt{3}i} \quad \underline{Ans}$$

Q.7 Multiply $z_1 = 2(\cos 60^\circ + i\sin 60^\circ)$ and $z_2 = 5(\cos 90^\circ + i\sin 90^\circ)$ and express in $x+iy$ form.

Sol:- Given that

$$Z_1 = 2(\cos 60^\circ + i\sin 60^\circ) \;\&\; Z_2 = 5(\cos 90^\circ + i\sin 90^\circ)$$

The product of two complex number in polar form.

$$Z_1\cdot Z_2 = r_1\cdot r_2\left[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right]$$

put $r_1 = 2, \theta_1 = 60^\circ \;\&\; r_2 = 5, \theta_2 = 90^\circ$

$$Z_1\cdot Z_2 = (2)(5)\left[\cos(60^\circ+90^\circ) + i\sin(60^\circ+90^\circ)\right]$$

$$= 10\left[\cos 150^\circ + i\sin 150^\circ\right]$$

$$= 10\left[-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right] = 10\cdot\frac{1}{2}\left(-\sqrt{3}+i\right)$$

$$Z_1\cdot Z_2 = 5\left(-\sqrt{3}+i\right)$$

$$\boxed{Z_1\cdot Z_2 = -5\sqrt{3}+5i} \quad \underline{Ans}$$

Q.8 Find the modulus & argument of $z = -2-2i$

Sol:- Given that

$$z = -2-2i$$

$$|z| = \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = \sqrt{4\times 2}$$

$$\boxed{|z| = 2\sqrt{2}}$$

Hence, Modulus of z is $2\sqrt{2}$

$$Arg(z) = Tan^{-1}\left|\frac{y}{x}\right|$$

$$Arg(z) = Tan^{-1}\left|\frac{-2}{-2}\right| = Tan^{-1}(1)$$

$$Arg(z) = \frac{\pi}{4}$$

As $z = -2-2i$ lie in Q-III, then

Req. $Arg(z) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$ or

$$Arg(z) = \frac{5\pi}{4} + 2n\pi, \; n \in Z$$

Q.9 Write the equation $\arg(\bar{z}-2+i) = \frac{2\pi}{3}$ in Cartesian form, if $z = x+iy$

Sol:- Given that

$$z = x+iy \Rightarrow \bar{z} = x-iy$$

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$$Arg(\bar{z}-2+i) = \frac{2\pi}{3}$$

$$Arg(x-iy-2+i) = \frac{2\pi}{3} \Rightarrow Arg(x-2+i(1-y)) = \frac{2\pi}{3}$$

$$Tan^{-1}\left|\frac{1-y}{x-2}\right| = \frac{2\pi}{3} \Rightarrow \left(\frac{1-y}{x-2}\right) = Tan\frac{2\pi}{3}$$

$$\frac{1-y}{x-2} = -\sqrt{3} \Rightarrow 1-y = (x-2)(-\sqrt{3})$$

$$1-y = -\sqrt{3}x + 2\sqrt{3}$$

$$\boxed{\sqrt{3}x - y + 1 - 2\sqrt{3} = 0}$$ Req. Cartesian form

10. If $z = x+iy$ and $Arg\left(\frac{\bar{z}-1+2i}{\bar{z}+1-2i}\right) = \frac{9\pi}{4}$, Show that $x^2+y^2-4x+2y-5=0$

Sol:- Given that

$$z = x+iy \Rightarrow \bar{z} = x-iy$$

$$Arg\left(\frac{\bar{z}-1+2i}{\bar{z}+1-2i}\right) = \frac{9\pi}{4}$$

As $Arg\left(\frac{z_1}{z_2}\right) = Arg\, z_1 - Arg(z_2)$

$$Arg(\bar{z}-1+2i) - Arg(\bar{z}+1-2i) = \frac{9\pi}{4}$$

put $\bar{z} = x-iy$

$$Arg(x-iy-1+2i) - Arg(x-iy+1-2i) = \frac{9\pi}{4}$$

$$\operatorname{Arg}(x-1+i(2-y)) - \operatorname{Arg}(x+1-(2+y)i) = \frac{9\pi}{4}$$

$$\tan^{-1}\left|\frac{2-y}{x-1}\right| - \tan^{-1}\left|\frac{-(2+y)}{x+1}\right| = \frac{9\pi}{4}$$

$$\tan^{-1}\left(\frac{2-y}{x-1}\right) - \tan^{-1}\left(\frac{-2-y}{x+1}\right) = \frac{9\pi}{4}$$

$$\because \tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

$$\tan^{-1}\left[\frac{\frac{2-y}{x-1} - \frac{(2-y)}{x+1}}{1+\left(\frac{2-y}{x-1}\right)\left(\frac{2-y}{x+1}\right)}\right] = \frac{9\pi}{4}$$

$$\tan^{-1}\left[\frac{\frac{(2-y)(x+1)+(2+y)(x-1)}{\cancel{(x-1)(x+1)}}}{\frac{(x-1)(x+1)-(2-y)(2+y)}{\cancel{(x-1)(x+1)}}}\right] = \frac{9\pi}{4}$$

$$\Rightarrow \frac{-y+2x+2-xy+(2x-2+xy-y)}{x^2-1-(2^2-y^2)} = \tan\frac{9\pi}{4}$$

$$\frac{-y+2x\cancel{+2}\cancel{-xy}+2x\cancel{-2}\cancel{+xy}-y}{x^2+y^2-4-1} = 1$$

$$\frac{4x-2y}{x^2+y^2-5} = 1 \Rightarrow 4x-2y = x^2+y^2-5$$

$$\Rightarrow \boxed{x^2+y^2-4x+2y-5=0}$$ Which is req.

Q.11 If $z = x+iy$ and $\arg(z-2-3i) - \arg(z+2+3i) = 2\pi$

Show that $2y = 3x$

Sol:- Given that $z = x+iy$

$$Arg(z-2-3i) - Arg(z+2+3i) = 2\pi$$

$$Arg(x+iy-2-3i) - Arg(x+iy+2+3i) = 2\pi$$

$$Arg(x-2+(y-3)i) - Arg(x+2+i(y+3)) = 2\pi$$

$$\tan^{-1}\left(\frac{y-3}{x-2}\right) - \tan^{-1}\left(\frac{y+3}{x+2}\right) = 2\pi$$

As $\tan^{-1}(A) - \tan^{-1}(B) = \tan^{-1}\left[\frac{A-B}{1+AB}\right]$

$$\tan^{-1}\left[\frac{\frac{y-3}{x-2} - \frac{y+3}{x+2}}{1+\left(\frac{y-3}{x-2}\right)\left(\frac{y+3}{x+2}\right)}\right] = 2\pi$$

$$\Rightarrow \frac{\frac{(y-3)(x+2)-(y+3)(x-2)}{\cancel{(x-2)(x+2)}}}{\frac{(x-2)(x+2)+(y-3)(y+3)}{\cancel{(x-2)(x+2)}}} = \tan(2\pi)$$

$$\frac{xy+2y-3x-6-(xy+3x-2y-6)}{(x-2)(x+2)+(y-3)(y+3)} = 0$$

$$\Rightarrow \cancel{xy}+2y-3x-\cancel{6}-\cancel{xy}-3x+2y+\cancel{6} = 0$$

$$4y-6x=0 \Rightarrow 2y-3x=0 \Rightarrow \boxed{2y=3x}$$

Q.12 Solve the equation $|z-2i| = |\bar{z}+2|$ for $z = x+iy$

Sol:- Given that

$$|z-2i| = |\bar{z}+2|$$

put $z = x+iy$ & $\bar{z} = x-iy$

$$|x+iy-2i| = |x-iy+2|$$

$$|x+i(y-2)| = |x+2-iy|$$

$$\sqrt{(x)^2+(y-2)^2} = \sqrt{(x+2)^2+(-y)^2}$$

$$\sqrt{x^2+y^2+4-4y} = \sqrt{x^2+4+4x+y^2}$$

Taking Square on b/s

$$\Rightarrow x^2+y^2+4-4y = x^2+4+4x+y^2$$

$$\Rightarrow \cancel{x^2}+\cancel{4}+4x+\cancel{y^2}-\cancel{x^2}-\cancel{y^2}-\cancel{4}+4y = 0$$

$$4x+4y = 0 \Rightarrow 4(x+y) = 0$$

$$\Rightarrow \boxed{x+y=0} \quad \text{OR} \quad \boxed{y=-x}$$

Q.13 For $z = x+iy$, solve the equation

$$|5z+4+i| = |5\bar{z}-3+2i|.$$

Sol:- Given that

$$|5z+4+i| = |5\bar{z}-3+2i|$$

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put $z = x+iy \Rightarrow \bar{z} = x-iy$ we get

$|5(x+iy)+4+i| = |5(x-iy)-3+2i|$

$|5x+5yi+4+i| = |5x-5yi-3+2i|$

$|5x+4+i(5y+4)| = |5x-3+i(2-5y)|$

$\Rightarrow \sqrt{(5x+4)^2+(5y+4)^2} = \sqrt{(5x-3)^2+(2-5y)^2}$

Taking Sq. on b/s we get

$\Rightarrow (5x+4)^2+(5y+4)^2 = (5x-3)^2+(2-5y)^2$

$\cancel{25x^2}+16+40x+\cancel{25y^2}+16+40y = \cancel{25x^2}+9-30x+4+\cancel{25y^2}-20y$

$40x+40y+32 = 13-30x-20y$

$40x+40y+32-13+30x+20y=0$

$$\boxed{70x+60y+19=0}$$

Q.14 Determine the set of points $z = x+iy$ that satisfy the equation $|3\bar{z}-2+i| = |3z+i|$

Sol:- Given that

$|3\bar{z}-2+i| = |3z+i|$

put $z = x+iy$ & $\bar{z} = x-iy$ we get

$|3(x-iy)-2+i| = |3(x+iy)+i|$

$|3x-3yi-2+i| = |3x+i3y+i|$

$$|3x-2+i(1-3y)| = |3x+i(1+3y)|$$

$$\Rightarrow \sqrt{(3x-2)^2+(1-3y)^2} = \sqrt{(3x)^2+(1+3y)^2}$$

Taking Square on b/s we get

$$\Rightarrow (3x-2)^2+(1-3y)^2 = (3x)^2+(1+3y)^2$$

$$\cancel{9x^2}+4-12x+\cancel{1}+\cancel{9y^2}-6y = \cancel{9x^2}+\cancel{1}+\cancel{9y^2}+6y$$

$$-12x+4 = 6y+6y$$

$$\Rightarrow 12y = -12x+4 \qquad \div\text{ing by } 12$$

$$y = -\frac{12}{12}x+\frac{4}{12}$$

$$\boxed{y = \frac{1}{3}-x}$$

Hence, The set of points z that satisfy the equation of the line $y = \frac{1}{3}-x$.

Q.15 An AC Source Supplies a voltage of $V = 120\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ volts to a circuit with impendance $z = \frac{1+i\sqrt{3}}{2}$ ohms. Calculate the current in polar form.

Sol:— Given that

$$\text{voltage} = v = 120\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) \text{Volts}$$

$$\text{Impedance} = Z = \frac{1+\sqrt{3}i}{2} = \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \text{Ohm's}$$

1st we find polar form of Impedance

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$r = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{\frac{1+3}{4}}$$

$$r = \sqrt{\frac{4}{4}} = \sqrt{1}$$

$$\boxed{r = 1}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right)$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{2}{1}\right)$$

$$\theta = \tan^{-1}(\sqrt{3})$$

$$\theta = \pi/3$$

The polar form of Impedance is $Z = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

For AC Circuit

Ohm's Law interm of Impedance is

$$I \cdot Z = v \Rightarrow I = \frac{v}{Z}$$

$$I = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

put $r_1 = 120$, $\theta_1 = \frac{\pi}{4}$ & $r_2 = 1$, $\theta_2 = \frac{\pi}{3}$

$$I = \frac{120}{1}\left[\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)\right]$$

$$= 120\left[\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right]$$

$$\boxed{I = 120\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)}$$ Which is req.

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Q.16 An AC Circuit has an impedance of $Z = 3-6i$ Ohm's and Connected to a voltage Source of $V = 90+30i$ Volts. Find the current in both rectangular & polar form.

Sol:— Given that

$$Z = 3-6i \quad \& \quad V = 90+30i$$

For AC Circuit

Ohm's law interm of Impedance is

$$I.Z = V \Rightarrow I = \frac{V}{Z}$$

$$I = \frac{90+30i}{3-6i} = \frac{\cancel{3}(30+10i)}{\cancel{3}(1-2i)} = \frac{30+10i}{1-2i}$$

Xing & ÷ing by $1+2i$

$$I = \frac{30+10i}{1-2i} \times \frac{1+2i}{1+2i} = \frac{30+60i+10i+20i^2}{1^2-2^2i^2}$$

$$I = \frac{30+70i+20(-1)}{1-4(-1)} = \frac{30+70i-20}{1+4}$$

$$I = \frac{10+70i}{5} = \frac{10}{5} + \frac{70}{5}i$$

$$\boxed{I = 2+14i}$$ Req. Rectangular form

For polar form

$$r = \sqrt{(2)^2+(14)^2}$$

$$r = \sqrt{4+196} = \sqrt{200}$$

$$r = \sqrt{100 \times 2}$$

$$\boxed{r = 10\sqrt{2}}$$

$$\theta = Tan^{-1}\left(\frac{14}{2}\right)$$

$$\theta = Tan^{-1}(7)$$

$$\boxed{\theta = 81.86^\circ}$$

$$I = 10\sqrt{2}\left[\cos 81.86^\circ + i\sin 81.86^\circ\right]$$

Req. polar form

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Q.17 Encrypt the word "CODE" by multiplying the complex encryption key $K = 2 - i$ Then decrypt it back to the original word.

Sol:- $K = 2 - i$

$C = 3 + 0i$, $O = 15 + 0i$, $D = 4 + 0i$, $E = 5 + 0i$

Encrypt the word CODE

$C' = (C)(K) = (3+0i)(2-i) = 3(2-i) = 6 - 3i$

$O' = (O)(K) = (15+0i)(2-i) = 15(2-i) = 30 - 15i$

$D' = (D)(K) = (4+0i)(2-i) = 4(2-i) = 8 - 4i$

$E' = (E)(K) = (5+0i)(2-i) = 5(2-i) = 10 - 5i$

Multiplicative inverse of encryption key

$$\frac{1}{K} = \frac{1}{2-i} = \frac{1}{2-i} \times \frac{2+i}{2+i} = \frac{2+i}{2^2 - i^2} = \frac{2+i}{4-(-1)} = \frac{2+i}{4+1}$$

$$\boxed{\frac{1}{K} = \frac{2+i}{5}}$$

Decrypt back to Original Word

$$C = \frac{C'}{K} = \frac{(6-3i)(2+i)}{5} = \frac{12+\cancel{6i}-\cancel{6i}-3i^2}{5} = \frac{12+3}{5} = \frac{15}{5} = 3$$

$$O = \frac{O'}{K} = \frac{(30-15i)(2+i)}{5} = \frac{1}{5}(60+\cancel{30i}-\cancel{30i}-15i^2) = \frac{1}{5}(60+15) = \frac{75}{5} = 15$$

$$D = \frac{D}{K} = \frac{1}{5}(8-4i)(2+i) = \frac{1}{5}(16+\cancel{8i}-\cancel{8i}-4i^2) = \frac{1}{5}(16+4) = \frac{1}{5}(20) = 4$$

$$E = \frac{E'}{K} = \frac{1}{5}(10-5i)(2+i) = \frac{1}{5}(20+\cancel{10i}-\cancel{10i}-5i^2) = \frac{1}{5}(20+5) = \frac{1}{5}(25) = 5$$

Hence The word "CODE" Encrypt & decrypt Successfully.

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Q.18 Consider the Complex encryption key $K = 3-3i$. Encrypt the word "QUIZ" and then recover the original word using the inverse of the key.

Sol:— $K = 3-3i$

$Q = 17,\ U = 21$

$I = 9,\ Z = 26$

Encrypt the word "QUIZ"

$$Q' = (Q)(K) = 17(3-3i) = 51-51i$$

$$U' = (U)(K) = 21(3-3i) = 63-63i$$

$$I' = (I)(K) = 9(3-3i) = 27-27i$$

$$Z' = (Z)(K) = 26(3-3i) = 78-78i$$

Multiplicative Inverse of encryption key.

$$\frac{1}{K} = \frac{1}{3-3i} = \frac{1}{3(1-i)} = \frac{1}{3(1-i)} \times \frac{1+i}{1+i} = \frac{1+i}{3(1-i^2)} = \frac{1+i}{3(1+1)} = \frac{1+i}{3(2)}$$

$$\boxed{\frac{1}{K} = \frac{1+i}{6}}$$

Decrypt the back to original word

$$Q = \frac{Q'}{K} = \frac{1}{6}(51-51i)(1+i) = \frac{1}{6}(51+\cancel{51i}-\cancel{51i}-51i^2) = \frac{1}{6}(51+51) = \frac{102}{6} = 17$$

$$U = \frac{U'}{K} = \frac{1}{6}(63-6i)(1+i) = \frac{1}{6}(63+\cancel{63i}-\cancel{63i}-6\ ) = \frac{1}{6}(63+63) = \frac{126}{6} = 21$$

$$I = \frac{I'}{K} = \frac{1}{6}(27-27i)(1+i) = \frac{1}{6}(27+\cancel{27i}-\cancel{27i}-27i^2) = \frac{1}{6}(27+27) = \frac{54}{6} = 9$$

$$Z = \frac{Z'}{K} = \frac{1}{6}(78-78i)(1+i) = \frac{1}{6}(78+\cancel{78i}-\cancel{78i}-78i^2) = \frac{1}{6}(78+78) = \frac{156}{6} = 26$$

Hence the word QUIZ encrypt & decrypt successfull.

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Q.19 Encrypt the word "CLASS" by adding the complex encryption key $K = -3+4i$. Then decrypt it back to original word.

Sol:-

$$K = -3+4i$$

$$C = 3+0i, \quad L = 12+0i, \quad A = 1+0i, \quad S = 19+0i$$

Encrypt the word "CLASS"

$$C' = C+K = 3-3+4i = 4i$$

$$L' = L+K = 12-3+4i = 9+4i$$

$$A' = A+K = 1-3+4i = -2+4i$$

$$S' = S+K = 19-3+4i = 16+4i$$

$$S' = S+K = 19-3+4i = 16+4i$$

Decrypt back to original word

$$C = C'-K = 4i-(-3+4i) = \cancel{4i}+3-\cancel{4i} = 3$$

$$L = L'-K = 9+4i-(-3+4i) = 9+\cancel{4i}+3-\cancel{4i} = 12$$

$$A = A'-K = -2+4i-(-3+4i) = -2+\cancel{4i}+3-\cancel{4i} = 1$$

$$S = S'-K = 16+4i-(-3+4i) = 16+\cancel{4i}+3-\cancel{4i} = 19$$

$$S = S'-K = 16+4i-(-3+4i) = 16+\cancel{4i}+3-\cancel{4i} = 19$$

Hence the word "CLASS" encrypt & decrypt successfully

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