

Exercise #3.2Q.1 Solve the following equations:

i) $\frac{1}{3x} + \frac{4x}{6} = 1, x \neq 0$

Sol: $\frac{1}{3x} + \frac{4x}{6} = 1$

$$\frac{1}{3x} + \frac{2x}{3} = 1$$

$$\frac{1+2x^2}{3x} = 1$$

$$\Rightarrow 1+2x^2 = 3x$$

$$2x^2 - 3x + 1 = 0$$

$$2x^2 - 2x - x + 1 = 0$$

$$2x(x-1) - 1(x-1) = 0$$

$$(x-1)(2x-1) = 0$$

$$x-1=0 \quad \vee \quad 2x-1=0$$

$$x=1$$

$$x=\frac{1}{2}$$

$$S.S = \left\{ 1, \frac{1}{2} \right\}$$

ii) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}, x \neq -1, 0$

Sol: $\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}$

$$\frac{x^2 + (x+1)^2}{x(x+1)} = \frac{5}{2}$$

$$\frac{x^2 + x^2 + 1 + 2x}{x^2 + x} = \frac{5}{2}$$

$$\Rightarrow 2(2x^2 + 2x + 1) = 5(x^2 + x)$$

$$4x^2 + 4x + 2 = 5x^2 + 5x$$

$$\Rightarrow 5x^2 + 5x - 4x^2 - 4x - 2 = 0$$

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x+2) - 1(x+2) = 0$$

$$(x+2)(x-1) = 0$$

$$x+2=0 \quad \wedge \quad x-1=0$$

$$x=-2$$

$$x=1$$

$$S.S = \{-2, 1\}$$

Ans

iii) $\frac{1}{x+1} + \frac{2}{x+2} = \frac{7}{x+5}, x \neq -1, -2, -5$

Sol: $\frac{1}{x+1} + \frac{2}{x+2} = \frac{7}{x+5}$

$$\frac{x+2 + 2x+2}{(x+1)(x+2)} = \frac{7}{x+5}$$

$$\frac{3x+4}{x^2+2x+2} = \frac{7}{x+5}$$

$$\Rightarrow (x+5)(3x+4) = 7(x^2+3x+2)$$

$$3x^2+4x+15x+20 = 7x^2+21x+14$$

$$3x^2+19x+20 = 7x^2+21x+14$$

$$\Rightarrow 7x^2+21x+14-3x^2-19x-20=0$$

$$4x^2+2x-6=0$$

$$\Rightarrow 2x^2+x-3=0$$

$$2x^2-2x+3x-3=0$$

$$2x(x-1)+3(x-1)=0$$

$$(x-1)(2x+3)=0$$

$$x-1=0 \quad \& \quad 2x+3=0$$

$$\boxed{x=1} \Rightarrow \boxed{x=-\frac{3}{2}}$$

$$\boxed{S.S = \left\{-\frac{3}{2}, 1\right\}} \quad \text{Ans}$$

$$\text{Ex) } \frac{a}{ax-1} + \frac{b}{bx-1} = a+b, x \neq \frac{1}{a}, \frac{1}{b}$$

$$\text{Sol:- } \frac{a}{ax-1} + \frac{b}{bx-1} = a+b$$

$$\left[\frac{a}{ax-1} - b \right] + \left[\frac{b}{bx-1} - a \right] = 0$$

$$\frac{a-b(ax-1)}{ax-1} + \frac{b-a(bx-1)}{bx-1} = 0$$

$$\frac{a-abx+b}{ax-1} + \frac{b-abx+a}{bx-1} = 0$$

$$\frac{a-abx+b}{ax-1} + \frac{a-abx+b}{bx-1} = 0$$

$$(a-abx+b) \left(\frac{1}{ax-1} + \frac{1}{bx-1} \right) = 0$$

$$a-abx+b=0 \quad \& \quad \frac{1}{ax-1} + \frac{1}{bx-1} = 0$$

$$a+b=abx \quad \left| \quad \frac{bx-1+ax-1}{(ax-1)(bx-1)} = 0 \right.$$

$$\Rightarrow \boxed{x = \frac{a+b}{ab}} \quad \Rightarrow (a+b)x - 2 = 0$$

$$(a+b)x = 2$$

$$\boxed{x = \frac{2}{a+b}}$$

$$\boxed{S.S = \left\{ \frac{a+b}{ab}, \frac{2}{a+b} \right\}}$$

$$\text{v) } \frac{x+1}{x-1} + \frac{x-1}{x+1} = 2$$

$$\text{Sol:- } \frac{x+1}{x-1} + \frac{x-1}{x+1} = 2$$

$$\frac{(x+1)^2 + (x-1)^2}{(x-1)(x+1)} = 2$$

$$\frac{x^2+1+2x+x^2+1-2x}{x^2-1} = 2$$

$$2x^2 + 2 = 2x - 2$$

$$2 \neq 2 \quad (\text{Not possible})$$

$$\boxed{S.S = \{ \}} \quad \text{Ans}$$

$$vi) \quad 3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} = 2$$

$$\text{Sol:- } 3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} = 2$$

$$3(x^2 + 5x) - 2\sqrt{x^2 + 5x + 1} = 2$$

$$\text{put } x^2 + 5x = y$$

$$3y - 2\sqrt{y+1} = 2$$

$$\Rightarrow 3y - 2 = 2\sqrt{y+1}$$

Taking square on both side

$$(3y - 2)^2 = (2\sqrt{y+1})^2$$

$$9y^2 + 4 - 12y = 4(y+1)$$

$$9y^2 - 12y + 4 = 4y + 4$$

$$9y^2 - 12y - 4y = 0 \Rightarrow 9y^2 - 16y = 0$$

$$y(9y - 16) = 0 \Rightarrow y = 0 \text{ or } 9y - 16 = 0$$

$$\boxed{y = 0}$$

$$\boxed{y = \frac{16}{9}}$$

Now $x^2 + 5x = y$ at $y = 0$

$$x^2 + 5x = 0$$

$x^2 + 5x = y$ at $y = \frac{16}{9}$

$$x^2 + 5x = \frac{16}{9}$$

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$$x(x+5) = 0$$

$$\boxed{x=0}$$

$$x+5=0$$

$$\boxed{x=-5}$$

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Checking

$$3(x^2+5x) - 2\sqrt{x^2+5x+1} = 2$$

For $x=0$ & $x=-5$ put $x^2+5x=0$

$$3(0) - 2\sqrt{0+1} = 2 \Rightarrow 0 - 2\sqrt{1} = 2$$

$$-2 \neq 2 \quad \text{Not true}$$

For $x=-16/3$ & $x=1/3$ put $x^2+5x=16/9$

$$3\left(\frac{16}{9}\right) - 2\sqrt{\frac{16}{9}+1} = 2 \Rightarrow \frac{16}{3} - 2\sqrt{\frac{16+9}{9}} = 2$$

$$\frac{16}{3} - 2\sqrt{\frac{25}{9}} = 2 \Rightarrow \frac{16}{3} - 2\left(\frac{5}{3}\right) = 2$$

$$\frac{16}{3} - \frac{10}{3} = 2 \Rightarrow \frac{16-10}{3} = 2$$

$$\frac{6}{3} = 2 \Rightarrow 2 = 2 \quad (\text{True})$$

$$\boxed{\text{Ans} = \left\{ \frac{1}{3}, -\frac{16}{3} \right\}}$$

$$9x^2+45x=16$$

$$9x^2+45x-16=0$$

$$9x^2+48x-3x-16=0$$

$$3x(3x+16)-1(3x+16)=0$$

$$(3x+16)(3x-1)=0$$

$$3x+16=0 \wedge 3x-1=0$$

$$\Rightarrow \boxed{x=-\frac{16}{3}}$$

$$\boxed{x=\frac{1}{3}}$$

$$\sqrt{2x+8} + \sqrt{x+5} = 7$$

$$\text{Sol: } \sqrt{2x+8} + \sqrt{x+5} = 7$$

$$\sqrt{2x+8} = 7 - \sqrt{x+5} \quad (\text{Taking sq.})$$

$$(\sqrt{2x+8})^2 = (7 - \sqrt{x+5})^2$$

$$2x+8 = 49 + x+5 - 14\sqrt{x+5}$$

$$2x+8 = 54 + x - 14\sqrt{x+5}$$

$$2x+8 - 54 - x = -14\sqrt{x+5}$$

$$x - 46 = -14\sqrt{x+5}$$

Again taking square on both side

$$(x - 46)^2 = (-14\sqrt{x+5})^2$$

$$x^2 + 2116 - 92x = 196(x+5)$$

$$x^2 + 2116 - 92x = 196x + 980$$

$$x^2 - 92x - 196x - 980 + 2116 = 0$$

$$x^2 - 288x + 1136 = 0$$

$$x^2 - 4x - 284x + 1136 = 0 \Rightarrow x(x-4) - 284(x-4) = 0$$

$$(x-4)(x-284) = 0$$

$$x-4 = 0$$

$$x-284 = 0$$

$$x = 4$$

$$x = 284$$

Checking

$$\sqrt{2x+8} + \sqrt{x+5} = 7$$

at $x = 4$

$$\sqrt{2(4)+8} + \sqrt{4+5} = 7 \Rightarrow \sqrt{16} + \sqrt{9} = 7 \Rightarrow 4+3=7$$
$$7=7 \text{ (True)}$$

at $x = 284$

$$\sqrt{(284)(2)+8} + \sqrt{284+5} = 7 \Rightarrow \sqrt{576} + \sqrt{289} = 7$$
$$\Rightarrow 24 + 17 = 7 \Rightarrow 41 \neq 7 \text{ (False)}$$

Hence

$$\boxed{S.S = \{4\}}$$

viii) $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

Sol:- $\sqrt{3x+4} = 2 + \sqrt{2x-4}$ (Taking square)

$$(\sqrt{3x+4})^2 = (2 + \sqrt{2x-4})^2$$

$$3x+4 = 4 + 2x - 4 + 4\sqrt{2x-4}$$

$$3x - 2x + 4 = 4\sqrt{2x-4}$$

$$x+4 = 4\sqrt{2x-4}$$

Again Taking square on b/s

$$(x+4)^2 = (4\sqrt{2x-4})^2$$

$$x^2 + 16 + 8x = 16(2x-4)$$

$$x^2 + 16 + 8x = 32x - 64$$

$$x^2 + 16 + 8x - 32x + 64 = 0 \Rightarrow x^2 - 24x + 80 = 0$$

$$x^2 - 20x - 4x + 80 = 0 \Rightarrow x(x - 20) - 4(x - 20) = 0$$

$$(x - 20)(x - 4) = 0$$

$$x - 20 = 0 \quad \vee \quad x - 4 = 0$$

$$\boxed{x = 20}$$

$$\boxed{x = 4}$$

Checking

$$\sqrt{3x+4} = 2 + \sqrt{2x-4}$$

at $x = 20$

$$\sqrt{3(20)+4} = 2 + \sqrt{2(20)-4} \Rightarrow \sqrt{64} = 2 + \sqrt{36}$$

$$8 = 2 + 6 \Rightarrow 8 = 8 \text{ (True)}$$

at $x = 4$

$$\sqrt{3(4)+4} = 2 + \sqrt{2(4)-4} \Rightarrow \sqrt{16} = 2 + \sqrt{4}$$

$$4 = 2 + 2 \Rightarrow 4 = 4 \text{ (True)}$$

$$\boxed{\text{S.S} = \{20, 4\}}$$

Ans

ix) $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Sol:- $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Taking Square on b/s

$$(\sqrt{x+7} + \sqrt{x+2})^2 = (\sqrt{6x+13})^2$$

$$x+7+x+2+\sqrt{(x+7)(x+2)} = 6x+13$$

$$2x+9+\sqrt{x^2+9x+14} = 6x+13$$

$$2\sqrt{x^2+9x+14} = 6x+13-2x-9$$

$$2\sqrt{x^2+9x+14} = 4x+4 \quad (\div \text{ by } 2)$$

$$\Rightarrow \sqrt{x^2+9x+14} = 2x+2 \quad (\text{Taking Square})$$

$$(\sqrt{x^2+9x+14})^2 = (2x+2)^2 \Rightarrow x^2+9x+14 = 4x^2+4+8x$$

$$\Rightarrow 4x^2+4+8x-x^2-9x-14=0$$

$$3x^2-x-10=0 \Rightarrow 3x^2-6x+5x-10=0$$

$$3x(x-2)+5(x-2)=0 \Rightarrow (x-2)(3x+5)=0$$

$$x-2=0$$

$$\text{or } 3x+5=0 \Rightarrow 3x=-5$$

$$\boxed{x=2}$$

$$\Rightarrow \boxed{x=-5/3}$$

Checking

$$\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

$$\text{at } x = -5/3$$

$$\sqrt{-5/3+7} + \sqrt{-5/3+2} = \sqrt{6(-5/3)+13} \Rightarrow \sqrt{-5+21} + \sqrt{-5+6} = \sqrt{-10+13}$$

$$\sqrt{16/3} + \sqrt{1/3} = \sqrt{3} \Rightarrow \frac{4}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \sqrt{3} \Rightarrow \frac{5}{\sqrt{3}} \neq \sqrt{3}$$

$$\text{at } x=2$$

$$\sqrt{2+7} + \sqrt{2+2} = \sqrt{6(2)+13} \Rightarrow \sqrt{9} + \sqrt{4} = \sqrt{25}$$

$$3+2 = 5 \Rightarrow 5 = 5 \text{ (True)}$$

$$\text{S.S} = \{2\}$$

Ans

$$x) \sqrt{x+5} - \sqrt{x-3} = 2$$

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$$\text{Sol: } - \sqrt{x+5} - \sqrt{x-3} = 2$$

$$\sqrt{x+5} = 2 + \sqrt{x-3}$$

(Taking Square)

$$(\sqrt{x+5})^2 = (2 + \sqrt{x-3})^2$$

$$\cancel{x+5} = 4 + \cancel{x-3} + 4\sqrt{x-3} \Rightarrow 5-1 = 4\sqrt{x-3}$$

$$4 = 4\sqrt{x-3} \Rightarrow \sqrt{x-3} = 1$$

Again Taking Square on b/s

$$(\sqrt{x-3})^2 = 1^2 \Rightarrow x-3 = 1 \Rightarrow x = 1+3$$

$$\boxed{x = 4}$$

Checking

$$\sqrt{x+5} - \sqrt{x-3} = 2$$

$$\text{at } x = 4$$

$$\sqrt{4+5} - \sqrt{4-3} = 2$$

$$\sqrt{9} - \sqrt{1} = 2 \Rightarrow 3-1 = 2$$

$$2 = 2 \text{ (True)}$$

$$\text{S.S} = \{4\}$$

Ans

Q.2 A farmer bought some sheep for Rs. 9000. If he had paid Rs. 100 less for each he would have got 3 sheep more for the same money. How many sheep did he buy, when the rate in each case is uniform?

Sol: Let Number of sheep = x

1st Condition Price of each sheep = y

$$\text{Total Price of sheep} = xy = 9000$$

$$\Rightarrow y = \frac{9000}{x} \quad \text{--- (1)}$$

2nd Condition

$$(y - 100)(x + 3) = 9000$$

$$\left(\frac{9000}{x} - 100\right)(x + 3) = 9000$$

$$\left(\frac{9000 - 100x}{x}\right)(x + 3) = 9000$$

$$(9000 - 100x)(x + 3) = 9000x$$

$$9000x + 27,000 - 100x^2 - 300x = 9000x$$

$$\Rightarrow 100x^2 + 300x - 27,000 = 0$$

Div by 100

$$x^2 + 3x - 270 = 0 \Rightarrow x^2 + 18x - 15x - 270 = 0$$

$$x(x + 18) - 15(x + 18) = 0$$

$$(x + 18)(x - 15) = 0$$

$$x + 18 = 0$$

$$x = -18$$

$$x - 15 = 0$$

$$x = 15$$

$$\boxed{\text{Req. No of sheep} = 15}$$

Ans

Q.3 A man sold his stock of eggs for Rs. 2400. If he had 2 dozen more, he would have got the same money by selling the whole for Rs. 0.50 per dozen cheaper. How many dozen eggs did he sell?

Sol:- Let Number of sell eggs in dozen = x

Price of each eggs = y

1st Condition:

$$\text{Total Price of eggs} = xy = 2400$$

$$\Rightarrow y = \frac{2400}{x} \quad \text{--- (1)}$$

2nd Condition:

$$(x+2)(y-0.50) = 2400$$

$$xy - 0.50x + 2y - 1 = 2400$$

$$\Rightarrow \cancel{2400} - 0.50x + 2\left(\frac{2400}{x}\right) - 1 = \cancel{2400}$$

$$\frac{4800}{x} - 0.50x - 1 = 0 \Rightarrow 4800 - 0.50x^2 - x$$

$$\Rightarrow 0.50x^2 + x - 4800 = 0 \quad (\text{using Q.F})$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(0.50)(-4800)}}{2(0.50)} = \frac{-1 \pm \sqrt{1 + 9600}}{1}$$

$$x = \frac{-1 \pm \sqrt{9601}}{1} = -1 \pm 97.98$$

$$x = -1 + 97.98$$

$$x = -1 - 97.98$$

$$x = 96.98 = 97$$

$$x = -98.98 \quad (\text{Not})$$

Req. Number of sell eggs in dozen = 97

Ans

Q.4 A cyclist travelled 48 km at a uniform speed. If he had travelled 2 km/hours slower, he would have taken 2 hours more to perform the journey. How long did he take to cover 48 km?

Sol:-

$$\text{Speed} = v \quad \& \quad \text{Time} = t = ?$$

1st Condition

$$\text{Total distance} = vt = 48$$

$$\Rightarrow v = \frac{48}{t}$$

2nd

Condition

$$(v-2)(t+2) = 48$$

$$vt + 2v - 2t - 4 = 48$$

$$\cancel{48} + 2\left(\frac{48}{t}\right) - 2t - 4 = \cancel{48}$$

$$\frac{96}{t} - 2t - 4 = 0 \Rightarrow 96 - 2t^2 - 4t = 0$$

$$\Rightarrow 2t^2 + 4t - 96 = 0 \Rightarrow t^2 + 2t - 48 = 0$$

$$t^2 + 8t - 6t - 48 = 0 \Rightarrow t(t+8) - 6(t+8) = 0$$

$$(t+8)(t-6) = 0$$

$$t+8 = 0 \quad \& \quad t-6 = 0$$

$$t = -8 \text{ (Not possible)} \Rightarrow t = 6$$

$$\boxed{t = 6 \text{ Take to cover 48 km}}$$

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Q.5 To do a piece of work, Abdullah takes 10 days more than Abdul Hadi. Together they finish the work in 12 days. How long would Abdul Hadi take to finish it alone?

Sol:- Let Abdullah take days = x ✓

Abdul Hadi take days = $x + 10$

To complete the work alone.

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Work done per Day

Abdullah work rate = $\frac{1}{x}$ ✓

Abdul Hadi work rate = $\frac{1}{x+10}$

Combined work rate = $\frac{1}{12}$

$$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{12} \Rightarrow \frac{x+10+x}{x^2+10x} = \frac{1}{12}$$

$$\frac{2x+10}{x^2+10x} = \frac{1}{12} \Rightarrow 24x+120 = x^2+10x$$

$$x^2+10x-24x-120=0 \Rightarrow x^2-14x-120=0$$

$$x^2-20x+6x-120=0 \Rightarrow x(x-20)+6(x-20)=0$$

$$(x-20)(x+6)=0 \Rightarrow x-20=0 \quad \text{✓} \quad x+6=0$$

$$x=20 \quad \text{✓} \quad x=-6$$

Req. Abdullah take days = 20

Q.6 The braking distance (in meters) of a car is modeled by $d(s) = 0.02s^2 + 0.1s$, where s is the speed of car in km/h. If the maximum safe braking distance is 50 meters, find the range of speed where braking is safe.

Sol:- The braking distance in meters is given by

$$d(s) = 0.02s^2 + 0.1s$$

The max. safe braking distance = 50 meter

$$\therefore d(s) \leq 50 \text{ m.}$$

$$\Rightarrow 0.02s^2 + 0.1s \leq 50 \Rightarrow 0.02s^2 + 0.1s - 50 = 0$$

$$0.02s^2 + 0.1s - 50 = 0$$

$$a = 0.02, \quad b = 0.1, \quad c = -50$$

using Quadratic Formula.

$$s = \frac{-0.1 \pm \sqrt{(0.1)^2 - 4(0.02)(-50)}}{2(0.02)} = \frac{-0.1 \pm \sqrt{0.01 + 4}}{0.04}$$

$$s = \frac{-0.1 \pm \sqrt{4.01}}{0.04} = \frac{-0.1 \pm 2.002}{0.04}$$

$$s = \frac{-0.1 + 2.002}{0.04}$$

$$s = \frac{-0.1 + 2.002}{0.04}$$

$$s = \frac{1.902}{0.04} = 47.56$$

$$s = \frac{-2.102}{0.04}$$

$$s = 47.56 \text{ km/h}$$

$$s = -52.55 \text{ (Not)}$$

The safe speed range is $0 \leq s \leq 47.56$

Q.7 A rocket follows the height function $h(t) = -5t^2 + 20t + 30$ where $h(t)$ is the height in meters and t is the time in seconds. Find the time interval during which the rocket is at least 40 meters above the ground.
 Sol: - Given that

$$h(t) = -5t^2 + 20t + 30$$

$$\text{Height of the rocket} = h(t) = 40\text{m}$$

$$\Rightarrow -5t^2 + 20t + 30 = 40$$

$$-5t^2 + 20t + 30 - 40 = 0 \Rightarrow -5t^2 + 20t - 10 = 0$$

Dividing by -5

$$t^2 - 4t + 2 = 0$$

(Using Quadratic formula)

$$t = \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2}$$

$$t = \frac{2(2 \pm \sqrt{2})}{2} = 2 \pm \sqrt{2} = 2 \pm 1.4142$$

$$t = 2 + 1.4142$$

$$t = 2 - 1.4142$$

$$t = 3.4142$$

$$t = 0.586$$

Required time interval are

$$0.586 \leq t \leq 3.414 \text{ s or } [0.586 \text{ s}, 3.414 \text{ s}]$$