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Chapter No. 05

Partial Fractions

Exercise # 5.1

Reblve the following into partial fractions:

Q.1 $\dfrac{2}{x^2-1}$

Sol:- $\dfrac{2}{x^2-1} = \dfrac{2}{(x-1)(x+1)}$ ——— (1 $\because a^2-b^2=(a+b)(a-b)$

Consider

$$\frac{2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \quad \text{——— (2}$$

Xing by $(x-1)(x+1)$ on b/s

$$(x-1)(x+1) \times \frac{2}{(x-1)(x+1)} = (x-1)(x+1) \times \frac{A}{x-1} + (x-1)(x+1) \times \frac{B}{x+1}$$

$$2 = A(x+1) + B(x-1) \quad \text{——— (3}$$

For A put $x = 1$ in eq. (3 we get

$$2 = A(1+1) + B(1-1) \Rightarrow 2 = 2A + B(0)$$

$$2 = 2A + 0 \Rightarrow 2A = 2 \Rightarrow A = 2/2$$

$$\boxed{A = 1}$$

For B put $x = -1$ in eq. (3 we get

$$2 = A(-1+1) + B(-1-1) \Rightarrow 2 = A(0) + B(-2)$$

$$2 = 0 - 2B \Rightarrow -2B = 2 \Rightarrow B = -2/2$$

$$\boxed{B = -1}$$

put $A = 1$ & $B = -1$ in eq. (2 we get

$$\frac{2}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{(-1)}{x+1}$$

$$\Rightarrow \boxed{\frac{2}{x^2-1} = \frac{1}{x-1} - \frac{1}{x+1}}$$ Ans

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Q.2 $\dfrac{a-b}{(x-a)(x-b)}$; $a \neq b$

Sol:- $$\frac{a-b}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b} \quad \text{---- (1}$$

xing by $(x-a)(x-b)$ on b/s

$$\Rightarrow a-b = A(x-b) + B(x-a) \quad \text{---- (2}$$

For A put $x = a$ in eq. (2 we get

$$a-b = A(a-b) + B(a-a)$$

$$a-b = A(a-b) + 0 \Rightarrow A = \frac{(a-b)}{(a-b)}$$

$$\boxed{A = 1}$$

For B put $x = b$ in eq. (2 we get

$$a-b = A(b-b) + B(b-a)$$

$$a-b = 0 - B(a-b) \Rightarrow B = -\frac{(a-b)}{(a-b)}$$

$$\boxed{B = -1}$$

put $A = 1$ & $B = -1$ in eq. (1 we get

$$\frac{a-b}{(x-a)(x-b)} = \frac{1}{x-a} + \frac{(-1)}{x-b}$$

$$\Rightarrow \boxed{\frac{a-b}{(x-a)(x-b)} = \frac{1}{x-a} - \frac{1}{x-b}} \quad \text{Ans}$$

Q.3 $\dfrac{x^2+1}{(x+1)(x-1)}$

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Sol:- $\dfrac{x^2+1}{(x+1)(x-1)} = \dfrac{x^2+1}{x^2-1}$

Since $\dfrac{x^2+1}{x^2-1}$ is improper fraction

$$\begin{array}{r|l} & 1 \\ \hline x^2-1 & x^2+1 \\ & \pm x^2 \mp 1 \\ \hline & 2 \end{array}$$

$$\frac{x^2+1}{x^2-1} = 1 + \frac{2}{x^2-1}$$

$$\Rightarrow \frac{x^2+1}{(x+1)(x-1)} = 1 + \frac{2}{(x+1)(x-1)} \quad \text{---(1}$$

Consider $\dfrac{2}{(x+1)(x-1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1}$ ---(2

Multiplying by $(x+1)(x-1)$ on b/s

$$\Rightarrow 2 = A(x-1) + B(x+1) \quad \text{---(3}$$

For A Put $x = -1$ in eq. (3 we get

$$2 = A(-1-1) + B(-1+1) \Rightarrow 2 = -2A + 0$$

$A = -2/2 \Rightarrow \boxed{A = 1}$

For B put $x = 1$ in eq. (3 we get.

$2 = A(1-1) + B(1+1)$

$2 = 0 + 2B \Rightarrow B = 2/2 \Rightarrow \boxed{B = 1}$

put $A = -1$ & $B = 1$ in eq. (2 we get

$$\frac{2}{(x+1)(x-1)} = \frac{-1}{x+1} + \frac{1}{x-1}$$ put in (1

$$\Rightarrow \boxed{\frac{x^2+1}{(x+1)(x-1)} = 1 - \frac{1}{x+1} + \frac{1}{x-1}}$$ Ans

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Q.4 $$\frac{2x+3}{(x+1)(x+2)(x+3)}$$

Sol:- $$\frac{2x+3}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$$ — (1

Xing by $(x+1)(x+2)(x+3)$ on b/s

$\Rightarrow 2x+3 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$ — (2

For A put $x = -1$ in eq. (2 we get

$\Rightarrow 2(-1)+3 = A(-1+2)(-1+3) + B(0) + C(0)$

$-2+3 = A(1)(2) + 0 + 0 \Rightarrow 1 = 2A \Rightarrow \boxed{A = \frac{1}{2}}$

For B put $x = -2$ in eq. (2 we get

$$2(-2)+3 = A(0) + B(-2+1)(-2+3) + C(0)$$

$$-4+3 = 0 + B(-1)(1) + 0 \Rightarrow -1 = -B$$

$$\Rightarrow \boxed{B = 1}$$

For C put $x = -3$ in eq. (2 we get

$$2(-3)+3 = A(0) + B(0) + C(-3+1)(-3+2)$$

$$-6+3 = 0 + 0 + C(-2)(-1)$$

$$-3 = 2C \Rightarrow \boxed{C = -3/2}$$

put $A = \frac{1}{2}$, $B = 1$ & $C = -3/2$ in eq. (1 we get

$$\frac{2x+3}{(x+1)(x+2)(x+3)} = \frac{1/2}{x+1} + \frac{1}{x+2} + \frac{-3/2}{x+3}$$

$$\Rightarrow \boxed{\frac{2x+3}{(x+1)(x+2)(x+3)} = \frac{1}{2(x+1)} + \frac{1}{x+2} - \frac{3}{2(x+3)}} \quad \text{Ans}$$

Q.5 $\dfrac{x^2+4x+5}{(x+1)(x^2+5x+6)}$

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Sol:- $x^2+5x+6 = x^2+3x+2x+6 = x(x+3)+2(x+3)$

$$x^2+5x+6 = (x+3)(x+2)$$

$$\frac{x^2+4x+5}{(x+1)(x^2+5x+6)} = \frac{x^2+4x+5}{(x+1)(x+2)(x+3)} \quad \text{(1}$$

Now $\frac{x^2+4x+5}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$ ——— (2

xing by $(x+1)(x+2)(x+3)$ we get ——— (3

$\Rightarrow x^2+4x+5 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$

For A put $x=-1$ in eq. (3 we get

$(-1)^2+4(-1)+5 = A(-1+2)(-1+3) + B(0) + C(0)$

$1-4+5 = A(1)(2) + 0 + 0 \Rightarrow 2 = 2A \Rightarrow A = \frac{2}{2}$

$\Rightarrow \boxed{A = 1}$

For B put $x=-2$ in eq. (3 we get

$(-2)^2+4(-2)+5 = A(0) + B(-2+1)(-2+3) + C(0)$

$4-8+5 = 0 + B(-1)(1) + 0 \Rightarrow 1 = -B$

$\Rightarrow \boxed{B = -1}$

For C put $x=-3$ in eq. (3 we get

$(-3)^2+4(-3)+5 = A(0) + B(0) + C(-3+1)(-3+2)$

$9-12+5 = 0 + 0 + C(-2)(-1) \Rightarrow 2 = 2C$

$C = 2/2 \Rightarrow \boxed{C = 1}$

put $A=1$, $B=-1$ & $C=1$ in eq. (2

$$\frac{x^2+4x+5}{(x+1)(x+2)(x+3)} = \frac{1}{x+1} + \frac{(-1)}{x+2} + \frac{1}{x+3}$$

$$= \frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{x+3}$$

$$\frac{x^2+4x+5}{(x+1)(x^2+5x+6)} = \frac{1}{x+1} - \frac{1}{x+2} + \frac{1}{x+3}$$ Ans

Q.6 $\dfrac{4x^3+5x^2-3x-2}{x^2-1}$

Sol:- As $\dfrac{4x^3+5x^2-3x-2}{x^2-1}$ is improper fraction.

$$\frac{4x^3+5x^2-3x-2}{x^2-1} = 4x+5+\frac{x+3}{x^2-1}$$

$$\begin{array}{r|l} & 4x+5 \\ \hline x^2-1 & 4x^3+5x^2-3x-2 \\ & \pm 4x^3 \mp 4x \\ \hline & 5x^2+x-2 \\ & \pm 5x^2 \mp 5 \\ \hline & x+3 \end{array}$$

$$\frac{4x^3+5x^2-3x-2}{x^2-1} = 4x+5+\frac{x+3}{(x+1)(x-1)} \quad \text{---(1}$$

Consider $\dfrac{x+3}{(x+1)(x-1)} = \dfrac{A}{x+1} + \dfrac{B}{x-1}$ ---(2

xing by $(x+1)(x-1)$ on b/s

$$\Rightarrow x+3 = A(x-1) + B(x+1) \quad \text{---(3}$$

For A put $x=-1$ in eq. (3 we get

$$-1+3 = A(-1-1) + B(0) \Rightarrow 2 = -2A+0$$

$$\Rightarrow A = -2/2 \Rightarrow \boxed{A=-1}$$

For B put $x=1$ in eq. (3 we get

$$1+3 = A(0) + B(1+1) \Rightarrow 4 = 0+2B$$

$$\Rightarrow B = 4/2 \Rightarrow \boxed{B=2}$$

put $A = -1$ & $B = 2$ in eq. (2 we get

$$\frac{x+3}{(x+1)(x-1)} = \frac{-1}{x+1} + \frac{2}{x-1}$$ put in (1

$$\boxed{\frac{4x^3+5x^2-3x-2}{x^2-1} = 4x+5-\frac{1}{x+1}+\frac{2}{x-1}}$$ Ans

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Q.7 $\dfrac{3x^2-12x+11}{(x-1)(x-2)(x-3)}$

Sol:- $\dfrac{3x^2-12x+11}{(x-1)(x-2)(x-3)} = \dfrac{A}{x-1}+\dfrac{B}{x-2}+\dfrac{C}{x-3}$ ——(1

x-ing by $(x-1)(x-2)(x-3)$ on b/s ——(2

$\Rightarrow 3x^2-12x+11 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$

For A put $x=1$ in eq. (2 we get

$3(1)^2-12(1)+11 = A(1-2)(1-3)+B(0)+C(0)$

$3-12+11 = A(-1)(-2)+0+0 \Rightarrow 2 = 2A \Rightarrow A = 2/2$

$\Rightarrow \boxed{A=1}$

For B put $x=2$ in eq. (2 we get

$3(2)^2-12(2)+11 = A(0)+B(2-1)(2-3)+C(0)$

$3(4)-24+11 = 0+B(1)(-1)+0 \Rightarrow 12-24+11 = -B$

$-1 = -B \Rightarrow \boxed{B=1}$

For C put $\boxed{x=3}$ in eq. (2 we get.

$3(3)^2-12(3)+11 = A(0) + B(0) + C(3-1)(3-2)$

$27-36+11 = 0+0+C(2)(1) \Rightarrow 2 = 2C \Rightarrow C = 2/2$

$$\boxed{C = 1}$$

put $A = 1$, $B = 1$ & $C = 1$ in eq. (1) we get

$$\boxed{\frac{3x^2-12x+11}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}}$$ Ans

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Q.8 $\dfrac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)}$

Sol:- $(x-1)(x-2)(x-3) = (x-1)(x^2-3x-2x+6)$

$= (x-1)(x^2-5x+6)$

$= x^3-5x^2+6x-x^2+5x-6$

$$\boxed{(x-1)(x-2)(x-3) = x^3-6x^2+11x-6}$$

$(x-4)(x-5)(x-6) = (x-4)(x^2-6x-5x+30)$

$= (x-4)(x^2-11x+30)$

$= x^3-11x^2+30x-4x^2+44x-120$

$$\boxed{(x-4)(x-5)(x-6) = x^3-15x^2+74x-120}$$

Now

$$\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = \frac{x^3-6x^2+11x-6}{x^3-15x^2+74x-120}.$$

As $\frac{x^3-6x^2+11x-6}{x^3-15x^2+74x-120}$ is improper fraction

$$x^3-15x^2+74x-120 \;\big|\; x^3-6x^2+11x-6 \;\big|\; 1$$

$$\pm x^3 \mp 15x^2 \pm 74x \mp 120$$

$$9x^2-63x+114$$

$$\frac{x^3-6x^2+11x-6}{x^3-15x^2+74x-120} = 1 + \frac{9x^2-63x+114}{x^3-15x^2+74x-120}$$

$$\Rightarrow \frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = 1 + \frac{-9x^2+63x-114}{(x-4)(x-5)(x-6)} \quad \text{---- (1}$$

$$\frac{9x^2-63x+114}{(x-4)(x-5)(x-6)} = \frac{A}{x-4} + \frac{B}{x-5} + \frac{C}{x-6} \quad \text{---- (2}$$

xing by $(x-4)(x-5)(x-6)$ on b/s ---- (3

$$\Rightarrow 9x^2-63x+114 = A(x-5)(x-6) + B(x-4)(x-6) + C(x-4)(x-5)$$

For A put $x=4$ in eq. (3 we get

$$9(4)^2-63(4)+114 = A(4-5)(4-6) + B(0) + C(0)$$

$$\Rightarrow 6 = A(-1)(-2) + 0 + 0 \Rightarrow 6 = 2A$$

$$\Rightarrow \boxed{A = 3}$$

For B put $x=5$ in eq. (3 we get

$$9(5)^2-63(5)+114 = A(0) + B(5-4)(5-6) + C(0)$$

$$\Rightarrow 24 = 0 + B(1)(-1) + 0 \Rightarrow 24 = -B$$

$$\boxed{B = -24}$$

For C put $x = 6$ in eq (3) we get

$$9(6)^2 - 63(6) + 114 = A(0) + B(0) + C(6-4)(6-5)$$

$$\Rightarrow 60 = 0 + 0 + C(2)(1) \Rightarrow 60 = 2C \Rightarrow C = 60/2$$

$$\boxed{C = 30}$$

put $A = 3$, $B = -24$ & $C = 30$ in eq. (2) we get

$$\frac{9x^2 - 63x + 114}{(x-4)(x-5)(x-6)} = \frac{3}{x-4} + \frac{(-24)}{x-5} + \frac{30}{x-6}$$

$$= \frac{3}{x-4} - \frac{24}{x-5} + \frac{30}{x-6}$$ put in (1)

$$\boxed{\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = 1 + \frac{3}{x-4} - \frac{24}{x-5} + \frac{30}{x-6}}$$ Ans

Q.9 $$\frac{x^2}{(x^2+a)(x^2+b)(x^2+c)}$$

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Sol:- put $y = x^2$

$$\frac{y}{(y+a)(y+b)(y+c)} = \frac{A}{y+a} + \frac{B}{y+b} + \frac{C}{y+c} \quad \text{---(1}$$

x'ing by $(y+a)(y+b)(y+c)$ on b/s

$$\text{---(2}$$

$$\Rightarrow y = A(y+b)(y+c) + B(y+a)(y+c) + C(y+a)(y+b)$$

For A put $y = -a$ in eq. (2 we get

$$-a = A(-a+b)(-a+c) + B(0) + C(0)$$

$$-a = -A(a-b)(c-a) \Rightarrow \boxed{A = \frac{a}{(a-b)(c-a)}}$$

For B put $y = -b$ in eq. (2) we get

$$-b = A(0) + B(-b+a)(-b+c) + C(0)$$

$$-b = -B(a-b)(b-c) \Rightarrow \boxed{B = \frac{b}{(a-b)(b-c)}}$$

For C put $y = -c$ in eq. (2) we get

$$-c = A(0) + B(0) + C(-c+a)(-c+b)$$

$$-c = -C(c-a)(b-c) \Rightarrow \boxed{C = \frac{c}{(c-a)(b-c)}}$$

Put $A = \frac{a}{(a-b)(c-a)}$, $B = \frac{b}{(a-b)(b-c)}$ & $C = \frac{c}{(c-a)(b-c)}$, $y = x^2$ in eq. (1) we get

$$\frac{x^2}{(x^2+a)(x^2+b)(x^2+c)} = \frac{\frac{a}{(a-b)(c-a)}}{x^2+a} + \frac{\frac{b}{(a-b)(b-c)}}{x^2+b} + \frac{\frac{c}{(c-a)(b-c)}}{x^2+c}$$

$$\boxed{\frac{x^2}{(x^2+a)(x^2+b)(x+c)} = \frac{a}{(a-b)(c-a)(x^2+a)} + \frac{b}{(x^2+b)(a-b)(b-c)} + \frac{c}{(c-a)(b-c)(x^2+c)}} \text{ Ans}$$

Q.10 $\frac{x+1}{(x-1)^2}$

Sol:- $\frac{x+1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$ ——— (1

xing by $(x-1)^2$ on b/s

$$\Rightarrow \quad x+1 = A(x-1) + B \quad \text{---- (2}$$

For B put $x = 1$ in eq. (2 we get

$$1+1 = A(1-1) + B \Rightarrow 2 = 0 + B$$

$$\boxed{B = 2}$$

Now $x+1 = Ax + B - A$

Comparing the the co-efficient of x, x^0

$$x \Rightarrow \boxed{1 = A}$$

put $A = 1$ & $B = 2$ in (1 we get

$$\boxed{\frac{x+1}{(x-1)^2} = \frac{1}{x-1} + \frac{2}{(x-1)^2}}$$ Ans

Q. 11 $\dfrac{x^2+x}{(x^2-1)^2}$

Sol:- $\dfrac{x^2+x}{(x^2-1)^2} = \dfrac{x(x+1)}{[(x+1)(x-1)]^2} = \dfrac{x(x+1)}{(x+1)^2(x-1)^2}$

$$\frac{x^2+x}{(x^2-1)^2} = \frac{x}{(x+1)(x-1)^2} \quad \text{---- (1}$$

$$\frac{x}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad \text{---- (2}$$

Xing by $(x+1)(x-1)^2$ on b/s

$$x = A(x-1)^2 + B(x+1)(x-1) + C(x+1) \quad \text{---(}$$

For A put $x = -1$ in eq.(we get

$$-1 = A(-1-1)^2 + B(0) + C(0)$$

$$-1 = A(-2)^2 + 0 + 0 \Rightarrow 4A = -1$$

$$\boxed{A = -1/4}$$

For C put $x = 1$ in eq.(we get

$$1 = A(0) + B(0) + C(1+1)$$

$$1 = 0 + 0 + 2C \Rightarrow 2C = 1$$

$$\boxed{C = 1/2}$$

Now $x = A(x^2+1-2x) + B(x^2-1) + C(x+1)$

$$x = Ax^2 + A - 2Ax + Bx^2 - B + Cx + C$$

Comparing the Co-efficient

$x^2 \Rightarrow A + B = 0$ put $A = -1/4$

$$-\frac{1}{4} + B = 0 \Rightarrow \boxed{B = 1/4}$$

put $A = -\frac{1}{4}$, $B = \frac{1}{4}$ & $C = \frac{1}{2}$ in eq.2

$$\frac{x}{(x+1)(x-1)^2} = \frac{-1/4}{x+1} + \frac{1/4}{x-1} + \frac{1/2}{(x-1)^2}$$

$$= \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$

$$\frac{x^2+x}{(x^2-1)^2} = \frac{1}{4(x+1)} + \frac{1}{4(x-1)} + \frac{1}{2(x-1)^2}$$ Ans

Q.12 $\dfrac{3x^2+4x-5}{(x-1)^3}$

Sol:- $\dfrac{3x^2+4x-5}{(x-1)^3} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3}$ ——— (1

Xing by $(x-1)^3$ on b/s

$\Rightarrow 3x^2+4x-5 = A(x-1)^2 + B(x-1) + C$ ——— (2

For C put $x=1$ in eq. (2 we get

$3(1)^2+4(1)-5 = A(0) + B(0) + C$

$\Rightarrow \boxed{C = 2}$

$3x^2+4x-5 = A(x^2+1-2x) + Bx - B + C$

$3x^2+4x-5 = Ax^2 + A - 2Ax + Bx - B + C$

Comparing the co-efficient

$x^2 \Rightarrow A = 3$

$x \Rightarrow 4 = -2A + B$ put $A = 3$

$4 = -2(3) + B \Rightarrow 4 = -6 + B$

$B = 4+6 \Rightarrow B = 10$

put $A=3$, $B=10$ & $C=2$ in eq. (1 we get

$$\frac{3x^2+4x-5}{(x-1)^3} = \frac{3}{x-1} + \frac{10}{(x-1)^2} + \frac{2}{(x-1)^3}$$ Ans

Q.13 $\dfrac{1}{x(x+1)^3}$

Sol:- $\dfrac{1}{x(x+1)^3} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2} + \dfrac{D}{(x+1)^3}$ —— (1

x ing by $x(x+1)^3$ on b/s

$1 = A(x+1)^3 + Bx(1+x)^2 + Cx(x+1) + Dx$ —— (2

For A put $x=0$ in eq. (2 we get

$1 = A(0+1)^3 + B(0) + (0) + D(0)$

$\Rightarrow \boxed{A = 1}$

For D put $x=-1$ in eq. (2 we get

$1 = A(0) + B(0) + C(0) + D(-1)$

$\Rightarrow -D = 1 \Rightarrow \boxed{D = -1}$

$1 = A(x^3+1+3x^2+3x) + Bx(x^2+1+2x) + Cx(x+1) + Dx$

$1 = Ax^3 + A + 3Ax^2 + 3Ax + Bx^3 + Bx + 2Bx^2 + Cx^2 + Cx + Dx$

Comparing the Co-efficients

$x^3 \Rightarrow 0 = A + B$ put $A = 1$

$0 = 1 + B \Rightarrow \boxed{B = -1}$

$x^2 \Rightarrow 0 = 3A + 2B + C \Rightarrow 0 = 3(1) + 2(-1) + C$

$\Rightarrow \boxed{C = -1}$

put $A=1$, $B=-1$, $C=-1$ & $D=-1$ in (1

$$\boxed{\frac{1}{x(x+1)^3} = \frac{1}{x} - \frac{1}{x+1} - \frac{1}{(x+1)^2} - \frac{1}{(x+1)^3}}$$ Ans

Q.14 $\dfrac{4x^2-3x+1}{(x+1)(x-1)^2}$

Sol:- $\dfrac{4x^2-3x+1}{(x+1)(x-1)^2} = \dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$ ——— (1

Xing by $(x+1)(x-1)^2$ on b/s

$\Rightarrow 4x^2-3x+1 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$ ——— (2

For A put $x=-1$ in eq. (2 we get

$4(-1)^2-3(-1)+1 = A(-1-1)^2 + B(0) + C(0)$

$\Rightarrow 8 = A(-2)^2 + 0 + 0 \Rightarrow 8 = 4A \Rightarrow A = 8/4$

$\Rightarrow \boxed{A = 2}$

For C put $x=1$ in eq. (2 we get

$4(1)^2-3(1)+1 = A(0) + B(0) + C(1+1)$

$2 = 0+0+2C \Rightarrow 2C = 2 \Rightarrow 2/2 = C$

$\Rightarrow \boxed{C = 1}$

Now $4x^2-3x+1 = A(x^2+1-2x) + B(x^2-1) + C(x+1)$

$4x^2-3x+1 = Ax^2+A-2Ax+Bx^2-B+Cx+C$

Comparing the co-efficient

$x^2 \Rightarrow 4 = A+B \Rightarrow 4 = 2+B \Rightarrow \boxed{B = 2}$

Put $A=2$, $B=2$ & $C=1$ in eq. (1 we get

$$\boxed{\frac{4x^2-3x+1}{(x+1)(x-1)^2} = \frac{2}{x+1} + \frac{2}{x-1} + \frac{1}{(x-1)^2}}$$ Ans

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Q.15 $\dfrac{12x^2-48}{(x-2)^2(x+2)^2}$

Sol:- $\dfrac{12x^2-48}{(x-2)^2(x+2)^2} = \dfrac{12(x^2-4)}{(x-2)^2(x+2)} = \dfrac{12(x-2)(x+2)}{(x-2)^2(x+2)^2}$

$$\frac{12x^2-48}{(x-2)^2(x+2)^2} = \frac{12}{(x-2)(x+2)} \quad \text{---- (1}$$

Now $\dfrac{12}{(x-2)(x+2)} = \dfrac{A}{x-2} + \dfrac{B}{x+2}$ ---- (2

Xing by $(x-2)(x+2)$ on b/s

$\Rightarrow 12 = A(x+2) + B(x-2)$ ---- (3

For A put $x=2$ in eq. (3 we get

$12 = A(2+2) + B(0) \Rightarrow 4A = 12 \Rightarrow A = 12/4$

$\Rightarrow \boxed{A=3}$

For B put $x=-2$ in eq. (3 we get

$12 = A(0) + B(-2-2) \Rightarrow -4B = 12 \Rightarrow B = \frac{-12}{4}$

$\Rightarrow \boxed{B=-3}$

Put $A=3$ & $B=-3$ in eq. (2 we get

$$\frac{12}{(x-2)(x+2)} = \frac{3}{x-2} + \frac{-3}{x+2} = \frac{3}{x-2} - \frac{3}{x+2}$$

Eq. (1 become

$$\boxed{\frac{12x^2-48}{(x-2)^2(x+2)^2} = \frac{3}{x-2} - \frac{3}{x+2}}$$

Ans