

Exercise #5.2

Resolve into partial fractions: **Sardar Abd - ul Qadeer Malak**

PhD (Mathematics Scholar)

HOD Math Department

0341-5838491

Q.1
$$\frac{2x^2 + 3x + 3}{(x+1)(x^2+1)}$$

Soln-
$$\frac{2x^2 + 3x + 3}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \quad (1)$$

Multiplying by $(x+1)(x^2+1)$ on both sides

$$\Rightarrow 2x^2 + 3x + 3 = A(x+1) + (Bx+C)(x+1) \quad (2)$$

For A put $x = -1$ in eq. (2) we get

$$2(-1)^2 + 3(-1) + 3 = A[(-1)^2 + 1] + (B(-1) + C)(-1+1)$$

$$\Rightarrow 2 = 2A \Rightarrow \boxed{A = 1}$$

$$2x^2 + 3x + 3 = Ax^2 + A + Bx^2 + Bx + Cx + C$$

Comparing the coefficients

$$x^2 \Rightarrow 2 = A + B \Rightarrow 2 = 1 + B \Rightarrow \boxed{B = 1}$$

$$x \Rightarrow 3 = B + C \Rightarrow 3 = 1 + C \Rightarrow \boxed{C = 2}$$

Put $A = 1$, $B = 1$ & $C = 2$ in eq. (1)

$$\frac{2x^2 + 3x + 3}{(x+1)(x^2+1)} = \frac{1}{x+1} + \frac{1 \cdot x + 2}{x^2+1}$$

$$\boxed{\frac{2x^2 + 3x + 3}{(x+1)(x^2+1)} = \frac{1}{x+1} + \frac{x+2}{x^2+1}}$$

Ans

Q.2

$$\frac{2x+1}{(x-2)(x^2+3x+5)}$$

Sardar Abd - ul Qadeer Malak

PhD (Mathematics Scholar)

HOD Math Department

0341-5838491

Sol:-

$$\frac{2x+1}{(x-2)(x^2+3x+5)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+3x+5} \quad \text{--- (1)}$$

Multiplying by $(x-2)(x^2+3x+5)$ on b/s

$$\frac{2x+1}{(x-2)(x^2+3x+5)} \times (x-2)(x^2+3x+5) = A(x^2+3x+5) + (Bx+C)(x-2)$$

$$\Rightarrow 2x+1 = A(x^2+3x+5) + (Bx+C)(x-2) \quad \text{--- (2)}$$

For A put $x=2$ in eq. (2) we get

$$2(2)+1 = A(2^2+3 \cdot 2+5) + (B \cdot 2+C)(0) \Rightarrow 5 = 15A$$

$$\Rightarrow \boxed{A = \frac{1}{3}}$$

$$2x+1 = Ax^2+3Ax+5A+Bx^2-2Bx+Cx-2C$$

Comparing the coefficients

$$x^2 \Rightarrow 0 = A+B \Rightarrow 0 = \frac{1}{3}+B \Rightarrow \boxed{B = -\frac{1}{3}}$$

$$x \Rightarrow 2 = 3A-2B+C \Rightarrow 2 = 3 \cdot \frac{1}{3} + \frac{2}{3} + C$$

$$2 = 1 + \frac{2}{3} + C \Rightarrow C = 2 - 1 - \frac{2}{3} = \frac{3-2}{3}$$

$$\boxed{C = \frac{1}{3}}$$

Put $A = \frac{1}{3}$, $B = -\frac{1}{3}$ & $C = \frac{1}{3}$ in eq. (1)

$$\frac{2x+1}{(x-2)(x^2+3x+5)} = \frac{\frac{1}{3}}{x-2} + \frac{-\frac{1}{3}x+\frac{1}{3}}{x^2+3x+5}$$

$$\frac{2x+1}{(x-2)(x^2+3x+5)} = \frac{1}{3(x-2)} + \frac{-\frac{1}{3}(x-1)}{x^2+3x+5}$$

$$\Rightarrow \boxed{\frac{2x+1}{(x-2)(x^2+3x+5)} = \frac{1}{3(x-2)} - \frac{(x-1)}{3(x^2+3x+5)}} \quad \text{Ans}$$

Sardar Abd - ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491

Q-3 $\frac{2x+32}{(x-2)(x^2+2)^2}$

Sol:- $\frac{2x+32}{(x-2)(x^2+2)^2} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2} + \frac{Dx+E}{(x^2+2)^2} \quad \text{--- (1)}$

Multiplying by $(x-2)(x^2+2)^2$ on b/s we get

$$\Rightarrow 2x+32 = A(x^2+2)^2 + (Bx+C)(x-2)(x^2+2) + (Dx+E)(x-2) \quad \text{--- (2)}$$

For A put $x=2$ in eq. (2) we get

$$2(2)+32 = A(2^2+2)^2 + (B \cdot 2 + C)(0) + (D \cdot 2 + E)(0)$$

$$4+32 = A(4+2)^2 + 0 + 0 \Rightarrow 36 = 36A \Rightarrow A = 36/36$$

$$\boxed{A = 1}$$

Now

$$2x+32 = A(x^4+4x^2+4) + (Bx+C)(x^3+2x^2-2x-4) + Dx^2-2Dx+Ex-2E$$

$$2x+32 = Ax^4+4Ax^2+Bx^4+2Bx^2-2Bx-4B+Cx^3+2Cx^2-2Cx-4C+Dx^2-2Dx+Ex-2E$$

Comparing the Co-efficients

$$x^4 \Rightarrow 0 = A + B \Rightarrow 0 = 1 + B \Rightarrow \boxed{B = -1}$$

$$x^3 \Rightarrow 0 = -2B + C \Rightarrow 0 = -2(-1) + C \Rightarrow \boxed{C = -2}$$

$$x^2 \Rightarrow 0 = 4A + 2B - 2C + D \Rightarrow 0 = 4(1) + 2(-1) + (-2)(-2) + D$$

$$D + 4 - 2 + 4 = 0 \Rightarrow D + 6 = 0 \Rightarrow \boxed{D = -6}$$

$$x \Rightarrow 2 = -4B + 2C - 2D + E \Rightarrow 2 = -4(-1) + 2(-2) - 2(-6) + E$$

$$E + 4 - 4 + 12 = 2 \Rightarrow E = 2 - 12 \Rightarrow \boxed{E = -10}$$

putting the values of A, B, C, D & E in eq. (1) we get

$$\frac{2x+32}{(x-2)(x^2+2)^2} = \frac{1}{x-2} + \frac{(-1)x-2}{x^2+2} + \frac{(-6)x-10}{(x^2+2)^2}$$

$$\boxed{\frac{2x+32}{(x-2)(x^2+2)^2} = \frac{1}{x-2} - \frac{(x+2)}{x^2+2} - \frac{2(3x+5)}{(x^2+2)^2}}$$

Ans

Q.4

$$\frac{3x^2+3}{x^3+1}$$

Sardar Abd-ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491

Sol:- $\frac{3x^2+3}{x^3+1} = \frac{3x^2+3}{(x+1)(x^2-x+1)} \quad \text{--- (1)}$

Now $\frac{3x^2+3}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \quad \text{--- (2)}$

Multiplying by $(x+1)(x^2-x+1)$ on b/s

$$\Rightarrow 3x^2+3 = A(x^2-x+1) + (Bx+C)(x+1) \quad \text{--- (3)}$$

For A put $x = -1$ in eq. (3) we get

$$3(-1)^2+3 = A(1+1+1) + (B(-1)+C)(0)$$

$$3+3 = 3A + 0 \Rightarrow A = 6/3$$

$$\boxed{A = 2}$$

$$3x^2 + 3 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C$$

Comparing the Co-efficients

$$x^2 \Rightarrow 3 = A + B \Rightarrow 3 = 2 + B \Rightarrow \boxed{B = 1}$$

$$x \Rightarrow 0 = -A + B + C \Rightarrow 0 = -2 + 1 + C \Rightarrow C - 1 = 0$$

$$\boxed{C = 1}$$

Putting the values of A, B & C in eq. (2)

$$\frac{3x^2 + 3}{(x+1)(x^2 - x + 1)} = \frac{2}{x+1} + \frac{(1)x + 1}{x^2 - x + 1}$$

$$\Rightarrow \boxed{\frac{3x^2 + 3}{x^3 + 1} = \frac{2}{x+1} + \frac{x+1}{x^2 - x + 1}} \quad \text{Ans}$$

Q.5

$$\frac{x^4}{x^4 + 2x^2 + 1}$$

Sardar Abd-ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491

As $\frac{x^4}{x^4 + 2x^2 + 1}$ is improper fraction

$$\frac{x^4}{x^4 + 2x^2 + 1} = 1 + \frac{-2x^2 - 1}{x^4 + 2x^2 + 1}$$

$$\begin{array}{r} x^4 + 2x^2 + 1 \overline{) \begin{array}{r} x^4 \\ + 2x^2 + 1 \\ \hline -2x^2 - 1 \end{array}} \end{array}$$

$$\frac{x^4}{x^4 + 2x^2 + 1} = 1 - \frac{(2x^2 + 1)}{(x^2 + 1)^2}$$

$$\frac{2x^2+1}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{C(x+D)}{(x^2+1)^2} \quad \text{--- (2)}$$

Multiplying by $(x^2+1)^2$ on b/s

$$2x^2+1 = (Ax+B)(x^2+1) + (x+D)$$

$$2x^2+1 = Ax^3 + Ax + Bx^2 + B + (x+D)$$

Comparing the Co-efficients

$$\begin{array}{l|l} x^3 \Rightarrow \boxed{A=0} & x^1 \Rightarrow \boxed{0=C} \\ x^2 \Rightarrow \boxed{2=B} & x^0 \Rightarrow \boxed{D=1} \end{array}$$

Putting the values of A, B, C & D in eq. (2)

$$\frac{2x^2+1}{(x^2+1)^2} = \frac{(0)x+2}{x^2+1} + \frac{(0)x+1}{(x^2+1)^2}$$

$$\frac{2x^2+1}{(x^2+1)^2} = \frac{2}{x^2+1} + \frac{1}{(x^2+1)^2} \quad \text{put in eq. (2)}$$

$$\boxed{\frac{x^4}{x^4+2x^2+1} = 1 - \frac{2}{x^2+1} - \frac{1}{(x^2+1)^2}}$$

Ans

Q.6

$$\frac{6x^4+40x^2}{(4-x^2)(x^2+4)^2}$$

Sardar Abd-ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491

Sol:- $\frac{6x^4+40x^2}{(4-x^2)(x^2+4)^2} = \frac{6x^4+40x^2}{(2-x)(2+x)(x^2+4)^2}$

$$\frac{6x^4 + 40x^2}{(2-x)(2+x)(x^2+4)^2} = \frac{A}{2-x} + \frac{B}{2+x} + \frac{Cx+D}{x^2+4} + \frac{Ex+F}{(x^2+4)^2} \quad (1)$$

Multiplying by $(2-x)(2+x)(x^2+4)$ on both sides

$$6x^4 + 40x^2 = A(2+x)(x^2+4) + B(2-x)(x^2+4) + (Cx+D)(2-x)(2+x)(x^2+4) + (Ex+F)(2-x)(2+x) \quad (2)$$

For A: put $x=2$ in eq. (2) we get

$$6(2)^4 + 40(2)^2 = A(2+2)(2^2+4) + B(0) + (C \cdot 2 + D)(0) + (E \cdot 2 + F)(0)$$

$$\Rightarrow 256 = A(4)(64) + 0 + 0 \Rightarrow 256A = 256$$

$$\boxed{A=1}$$

For B: put $x=-2$ in eq. (2) we get

$$6(-2)^4 + 40(-2)^2 = A(0) + B(2+2)(4+4) + (C(-2)+D)(0) + (E(-2)+F)(0)$$

$$256 = 0 + B(4)(64) + 0 + 0 \Rightarrow 256 = 256B$$

$$\boxed{B=1}$$

$$6x^4 + 40x^2 = A(2+x)(x^4+16+8x^2) + B(2-x)(x^4+16+8x^2) + (Cx+D)(4-x^2)(x^2+4) + (Ex+F)(4-x^2)$$

$$6x^4 + 40x^2 = A(2x^4 + 32 + 16x^2 + x^5 + 16x + 8x^3) + B(2x^4 + 32 + 16x^2 - x^5 - 16x - 8x^3) + (Cx+D)(16-x^4) + (Ex+F)(4-x^2)$$

$$6x^4 + 40x^2 = 2Ax^4 + 32A + 16Ax^2 + Ax^5 + 16Ax + 8Ax^3 + 2Bx^4 + 32B + 16Bx^2 - Bx^5 - 16Bx - 8Bx^3 + 16Cx - (Cx^5 + D)16 - Dx^4 + Ex^4 - Ex^2 + 4F - Fx^2$$

Comparing the Co-efficients

$$x^5 \Rightarrow 0 = A - B - C \Rightarrow 0 = 1 - 1 - C \Rightarrow \boxed{C = 0}$$

$$x^4 \Rightarrow 6 = 2A + 2B - D \Rightarrow 6 = 2(1) + 2(1) - D \Rightarrow D = 4 - 6$$

$$\boxed{D = -2}$$

$$x^3 \Rightarrow 0 = 8A - 8B - E \Rightarrow 0 = 8 - 8 - E \Rightarrow \boxed{E = 0}$$

$$x^2 \Rightarrow 40 = 16A + 16B - F \Rightarrow 40 = 16 + 16 - F \Rightarrow F = 32 - 40$$

$$\boxed{F = -8}$$

putting the values of A, B, C, D, E & F in eq. 2

$$\frac{6x^4 + 40x^2}{(2-x)(2+x)(x^2+4)} = \frac{1}{2-x} + \frac{1}{2+x} + \frac{(0)x-2}{x^2+4} + \frac{(0)x-8}{(x^2+4)^2}$$

$$\Rightarrow \boxed{\frac{6x^4 + 40x^2}{(4-x^2)(x^2+4)^2} = \frac{1}{2-x} + \frac{1}{2+x} - \frac{2}{x^2+4} - \frac{8}{(x^2+4)^2}} \quad \text{Ans}$$

Sardar Abd - ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abd - ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abd - ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abd - ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491

Sardar Abd - ul Qadeer Malak
PhD (Mathematics Scholar)
HOD Math Department
0341-5838491